

## Контрольная работа за 2 семестр.

Задание 1. Найти пределы.

- 1.1 а)  $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 2}{2x^2 + 4x + 1}$  б)  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{2x + 4}$
- в)  $\lim_{x \rightarrow 7} \frac{x^2 - 49}{2 - \sqrt{x - 3}}$  г)  $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$  д)  $\lim_{x \rightarrow 0} (1 - 4x)^{\frac{1-x}{x}}$
- 1.2 а)  $\lim_{x \rightarrow \infty} \frac{x^4 - 4x^2 + 1}{2x^4 - x^3 + 3}$  б)  $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 - x - 2}$
- в)  $\lim_{x \rightarrow 3} \frac{9 - x^2}{\sqrt{3x} - 3}$  г)  $\lim_{x \rightarrow 0} \frac{\sin^2\left(\frac{x}{2}\right)}{x^2}$  д)  $\lim_{x \rightarrow \infty} \left(x - \sqrt{x^2 - x + 1}\right)$
- 1.3 а)  $\lim_{x \rightarrow \infty} \frac{6x^3 - 4x^2 + 2}{5x^3 + 2x + 1}$  б)  $\lim_{x \rightarrow -1} \frac{x^2 + 2x + 1}{x + 1}$
- в)  $\lim_{x \rightarrow -2} \frac{x + 2}{\sqrt{2 - x} - \sqrt{x + 6}}$  г)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sqrt{x + 2} - \sqrt{2}}$  д)  $\lim_{x \rightarrow \infty} \left(1 + \frac{4}{x}\right)^{x+3}$
- 1.4 а)  $\lim_{x \rightarrow \infty} \frac{4x^2 - 3x + 2}{8x^2 - 4x + 1}$  б)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4}$
- в)  $\lim_{x \rightarrow 2} \frac{\sqrt{5 - x} - \sqrt{x + 1}}{x^2 - 3x + 2}$  г)  $\lim_{x \rightarrow \frac{1}{2}} \frac{\arcsin(1 - 2x)}{4x^2 - 1}$  д)  $\lim_{x \rightarrow \infty} x \cdot (\ln x - \ln(x + 2))$
- 1.5 а)  $\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 - 3}{4x^3 - 6x + 3}$  б)  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x \cdot \sin x}$
- в)  $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 9}$  г)  $\lim_{x \rightarrow 7} \frac{2 - \sqrt{x - 3}}{x^2 - 49}$  д)  $\lim_{x \rightarrow \infty} x \cdot (\ln(x + 3) - \ln x)$
- 1.6 а)  $\lim_{x \rightarrow \infty} \frac{14x^2 + 3x}{7x^2 + 2x - 8}$  б)  $\lim_{x \rightarrow -5} \frac{\sqrt{3x + 17} - \sqrt{2x + 12}}{x^2 + 8x + 15}$
- в)  $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$  г)  $\lim_{x \rightarrow 0} \frac{x \cdot \operatorname{tg} 3x}{\cos x - \cos^3 x}$  д)  $\lim_{x \rightarrow \infty} (3x + 5) \cdot (\ln(x + 5) - \ln x)$
- 1.7 а)  $\lim_{x \rightarrow \infty} \frac{4x^6 - x^3 + 2x}{2x^6 - 1}$  б)  $\lim_{x \rightarrow \infty} (x - 4) \cdot (\ln(2 - 3x) - \ln(5 - 3x))$
- в)  $\lim_{x \rightarrow -2} \frac{3x + 6}{x^3 + 8}$  г)  $\lim_{x \rightarrow 0} \frac{\cos x - \cos^5 x}{x^2}$  д)  $\lim_{x \rightarrow -3} \frac{\sqrt{x + 10} - \sqrt{4 - x}}{2x^2 - x - 21}$

1.8 a)  $\lim_{x \rightarrow \infty} \frac{3x^2 - 4x + 2}{6x^2 + 2x - 4}$       б)  $\lim_{x \rightarrow -2} \frac{\sqrt{2-x} - \sqrt{x+6}}{x^2 - x - 6}$

е)  $\lim_{x \rightarrow 3} \frac{9 - x^2}{x - 3}$       з)  $\lim_{x \rightarrow 0} \frac{\arctg 2x}{5x}$       д)  $\lim_{x \rightarrow 2} (2x - 3)^{\frac{3x}{x-2}}$

1.9 a)  $\lim_{x \rightarrow \infty} \frac{5x^3 - 6x - 3}{x^3 - 2x^2 + 4}$       б)  $\lim_{x \rightarrow \infty} (2x - 7) \cdot (\ln(3x + 4) - \ln 3x)$

е)  $\lim_{x \rightarrow 5} \frac{\sqrt{2x+1} - \sqrt{x+6}}{2x^2 - 7x - 15}$       з)  $\lim_{x \rightarrow 0} x \cdot \text{ctg } 5x$       д)  $\lim_{x \rightarrow -3} \frac{x+3}{x^2 - 9}$

1.10 a)  $\lim_{x \rightarrow \infty} \frac{8 - 2x + 5x^4}{2 + 3x^3 + x^4}$       б)  $\lim_{x \rightarrow \infty} (x - 4) \cdot (\ln(2 - 3x) - \ln(5 - 3x))$

е)  $\lim_{x \rightarrow -4} \frac{3 - \sqrt{x^2 - 7}}{2 - \sqrt{8 + x}}$       з)  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^2}}{\cos x - \cos^3 x}$       д)  $\lim_{x \rightarrow 3} \frac{x^2 + 4x - 21}{2x^2 - 7x + 3}$

1.11 a)  $\lim_{x \rightarrow \infty} \frac{x^2 - 5x + 1}{3x + 7}$       б)  $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3}$

е)  $\lim_{x \rightarrow 4} \frac{3 - \sqrt{5 + x}}{1 - \sqrt{5 - x}}$       з)  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x^2}$       д)  $\lim_{x \rightarrow \infty} \left( \frac{x-2}{x} \right)^{x-3}$

1.12 a)  $\lim_{x \rightarrow \infty} \frac{8x^2 + 4x - 5}{4x^2 - 3x + 2}$       б)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

е)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{\sqrt{5-x} - \sqrt{x+1}}$       з)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \cdot \sin x}$       д)  $\lim_{x \rightarrow 1} (2 - x)^{\frac{2x}{1-x}}$

1.13 a)  $\lim_{x \rightarrow \infty} \frac{3x^4 - 6x^2 + 2}{x^4 + 4x - 2}$       б)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3}$

е)  $\lim_{x \rightarrow 4} \frac{2x^2 - 9x + 4}{\sqrt{5-x} - \sqrt{x-3}}$       з)  $\lim_{x \rightarrow 0} \frac{\cos x - \cos^3 x}{x \cdot \sin 2x}$       д)  $\lim_{x \rightarrow 1} (2x - 1)^{\frac{2x}{1-x}}$

1.14 a)  $\lim_{x \rightarrow \infty} \frac{8x^4 - 4x^2 + 3}{2x^4 + 1}$       б)  $\lim_{x \rightarrow 2} (3 - x) \cdot (\ln(1 - x) - \ln(2 - x))$

е)  $\lim_{x \rightarrow 1} \frac{\sqrt{3+2x} - \sqrt{x+4}}{3x^2 - 4x + 1}$       з)  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 8x}$       д)  $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^3 + 1}$

1.15 a)  $\lim_{x \rightarrow \infty} \frac{3x^3 - 2x + 7}{9x^3 + 3x + 5}$       б)  $\lim_{x \rightarrow \infty} (4x - 3) \cdot (\ln(x + 2) - \ln(x - 1))$

е)  $\lim_{x \rightarrow 1} \frac{2 - \sqrt{5-x}}{3 - \sqrt{8+x}}$       з)  $\lim_{x \rightarrow 0} \frac{x \cdot \text{tg } 3x}{\cos x - \cos^3 x}$       д)  $\lim_{x \rightarrow -4} \frac{2x^2 + 7x - 4}{2x^2 + 13x + 20}$

1.16 a)  $\lim_{x \rightarrow \infty} \frac{4 + 5x^2 - 2x^5}{8 + 6x - x^5}$       б)  $\lim_{x \rightarrow \infty} (3x - 1) \cdot (\ln(4x - 1) - \ln(4x + 1))$

в)  $\lim_{x \rightarrow -3} \frac{5 - \sqrt{22 - x}}{1 - \sqrt{4 + x}}$       г)  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{2x \cdot \operatorname{tg} 2x}$       д)  $\lim_{x \rightarrow 2} \frac{2x^2 + x - 10}{x^2 - x - 2}$

1.17 a)  $\lim_{x \rightarrow \infty} \frac{3 - 7x^2 + 5x^3}{2 + 2x - x^3}$       б)  $\lim_{x \rightarrow \infty} (2x - 5) \cdot (\ln(2x + 4) - \ln(2x + 1))$

в)  $\lim_{x \rightarrow -5} \frac{\sqrt{9 + x} - 2}{\sqrt{4 - x} - 3}$       г)  $\lim_{x \rightarrow 0} \frac{1 - \cos 6x}{1 - \cos 2x}$       д)  $\lim_{x \rightarrow -3} \frac{x^2 + 10x + 21}{x^2 + 8x + 15}$

1.18 a)  $\lim_{x \rightarrow \infty} \frac{1 + 4x - x^4}{x + 3x^2 + 2x^4}$       б)  $\lim_{x \rightarrow \infty} (2x + 3) \cdot (\ln(x + 2) - \ln x)$

в)  $\lim_{x \rightarrow -4} \frac{\sqrt{x + 12} - \sqrt{4 - x}}{x^2 + 2x - 8}$       г)  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$       д)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2}$

1.19 a)  $\lim_{x \rightarrow \infty} \frac{2x^3 + 7x - 2}{6x^3 - 4x + 3}$       б)  $\lim_{x \rightarrow 1} (3 - 2x)^{\frac{x}{1-x}}$

в)  $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{\sqrt{x - 2} - \sqrt{4 - x}}$       г)  $\lim_{x \rightarrow 0} \frac{\sin^2\left(\frac{x}{4}\right)}{x^2}$       д)  $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x - 1}$

1.20 a)  $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 7}{5x^3 - x + 2}$       б)  $\lim_{x \rightarrow \infty} (3x + 2) \cdot (\ln(x + 1) - \ln x)$

в)  $\lim_{x \rightarrow 2} \frac{\sqrt{3x - 2} - 2}{\sqrt{2x + 5} - 3}$       г)  $\lim_{x \rightarrow 0} \frac{x^2 \cdot \operatorname{ctg} 2x}{\sin 3x}$       д)  $\lim_{x \rightarrow 5} \frac{3x^2 - 14x - 5}{x^2 - 2x - 15}$

1.21 a)  $\lim_{x \rightarrow \infty} \frac{2x^4 + 5x^2 - 3}{5x^4 - 2x^3 - 4}$       б)  $\lim_{x \rightarrow \infty} (x + 2) \cdot (\ln(2x + 1) - \ln(2x - 1))$

в)  $\lim_{x \rightarrow -1} \frac{\sqrt{2x + 3} - 1}{\sqrt{5 + x} - 2}$       г)  $\lim_{x \rightarrow 0} \frac{\cos x - \cos^5 x}{x^2}$       д)  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{2x^2 - x - 1}$

1.22 a)  $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{3x^2 + x - 5}$       б)  $\lim_{x \rightarrow \infty} (2x - 3) \cdot (\ln(x - 2) - \ln(x + 1))$

в)  $\lim_{x \rightarrow 5} \frac{1 - \sqrt{x - 4}}{2 - \sqrt{2x - 6}}$       г)  $\lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x}{x^2}$       д)  $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 10}{2x^2 + 9x + 10}$

1.23 a)  $\lim_{x \rightarrow \infty} \frac{4x^3 - 2x + 1}{2x^3 + 3x^2 - 2}$       б)  $\lim_{x \rightarrow \infty} (x - 5) \cdot (\ln(x - 3) - \ln x)$

в)  $\lim_{x \rightarrow -2} \frac{3 - \sqrt{x + 11}}{2 - \sqrt{6 + x}}$       г)  $\lim_{x \rightarrow 0} \frac{\sin x}{\arcsin x}$       д)  $\lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{2x^2 - 13x + 20}$

1.24 а)  $\lim_{x \rightarrow \infty} \frac{3x + 14x^2}{1 + 2x + 7x^2}$  б)  $\lim_{x \rightarrow \infty} (2x - 7) \cdot (\ln(x + 4) - \ln x)$   
 в)  $\lim_{x \rightarrow 4} \frac{1 - \sqrt{x - 3}}{2 - \sqrt{x}}$  г)  $\lim_{x \rightarrow 0} \frac{\arcsin 3x}{6x}$  д)  $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^2 + 8x + 15}$

1.25 а)  $\lim_{x \rightarrow \infty} \frac{3x^3 - 5x^2 + 2}{3x^3 + 5x^2 - x}$  б)  $\lim_{x \rightarrow \infty} (2x + 1) \cdot (\ln(x + 3) - \ln x)$   
 в)  $\lim_{x \rightarrow 3} \frac{\sqrt{2x + 3} - 3}{\sqrt{x - 2} - 1}$  г)  $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} 2x}{4x}$  д)  $\lim_{x \rightarrow -1} \frac{2x^2 + 3x + 1}{2x^2 + 5x + 3}$

1.26 а)  $\lim_{x \rightarrow \infty} \frac{8 - 2x + 5x^4}{2 + 3x^2 + x^4}$  б)  $\lim_{x \rightarrow \infty} (x + 5) \cdot (\ln(2x - 3) - \ln(2x + 3))$   
 в)  $\lim_{x \rightarrow 2} \frac{\sqrt{2x - 3} - 1}{\sqrt{2x + 5} - 3}$  г)  $\lim_{x \rightarrow 0} \frac{x}{\operatorname{arctg} 5x}$  д)  $\lim_{x \rightarrow 1} \frac{x^3 - x}{x^2 - 6x + 5}$

1.27 а)  $\lim_{x \rightarrow \infty} \frac{3x + 14x^3}{1 + 2x + 7x^3}$  б)  $\lim_{x \rightarrow \infty} (x + 3) \cdot (\ln(2 - 4x) - \ln(1 - 4x))$   
 в)  $\lim_{x \rightarrow 1} \frac{\sqrt{2x + 7} - 3}{\sqrt{x + 8} - 3}$  г)  $\lim_{x \rightarrow 0} \frac{\sin x}{\arcsin\left(\frac{x}{2}\right)}$  д)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - x}$

**Задание 2.** Найти первые производные функций.

2.1. а)  $y = 3x^5 - \frac{1}{x} + \sqrt[4]{x}$ ; е)  $y = \ln \operatorname{tg}(2x + 1)$ ;  
 б)  $y = \frac{\sin x}{\cos^2 x}$ ; ж)  $y = \frac{x^3}{(x - 2)^2}$ ;  
 в)  $y = (x + 1)^2 \cdot \cos 5x$ ; з)  $y = 2^{3x} + 7x^7 + e^{-x^2}$ ;  
 г)  $y = \operatorname{arctg}(e^{2x} + 3)$ ; и)  $y = 0,7^{\operatorname{ctg}^2 x}$ ;  
 д)  $y = \sqrt{x + \sqrt[3]{x}}$ ; к)  $y = x^{\arcsin x}$ .

2.2. а)  $y = 4x^7 + \frac{1}{x^2} - \sqrt{2x}$ ; е)  $y = x^2 \cdot \cos 7x$ ;  
 б)  $y = \frac{x + e^{3x+2}}{1 + \cos 3x}$ ; ж)  $y = \frac{x^2}{(x + 1)^2}$ ;  
 в)  $y = (x + 2) \cdot e^{-x^2}$ ; з)  $y = \ln^5 \sin x$ ;  
 г)  $y = \sin(3x^7 + 1) + 8x$ ; и)  $y = \arcsin e^{4x}$ ;  
 д)  $y = 2^{\operatorname{tg} x} + 3^{\cos 4x}$ ; к)  $y = (\sin 2x)^{\frac{1}{x}}$

2.3. а)  $y = 7x - \sqrt[3]{x} + \frac{1}{x^3}$ ; е)  $y = \sin^4 x + \cos^4 x$ ;  
 б)  $y = \sqrt[3]{\frac{1 + x^2}{1 - x^2}}$ ; ж)  $y = \ln \frac{x^2}{1 - x^2}$ ;

$$\text{в)} y = 3x \cdot \arcsin 2x;$$

$$\text{г)} y = \sqrt{x^2 + 1} + \sqrt[3]{x^3 + 1};$$

$$\text{д)} y = 3^{\text{ctg} x} + 8^{\cos 4x};$$

$$2.4. \text{ а)} y = 9x^2 + \frac{1}{2x^2} - \sqrt[3]{x};$$

$$\text{б)} y = \sqrt[3]{1 + x^3};$$

$$\text{в)} y = e^{\sin 5x} \cdot \ln x;$$

$$\text{г)} y = \ln \sin(2x + 5);$$

$$\text{д)} y = 0,9^{\cos^2 x};$$

$$2.5. \text{ а)} y = 3x^5 - \frac{1}{x^5} - \sqrt[5]{x};$$

$$\text{б)} y = 2\sqrt{4x + 3} - \frac{3}{\sqrt{x^3 + x + 1}};$$

$$\text{в)} y = (\ln x + 1)^2 \cdot \cos 2x;$$

$$\text{г)} y = \arcsin \sqrt{1 - 4x};$$

$$\text{д)} y = 5^{\text{tg} x} + 3^{\sin x};$$

$$2.6. \text{ а)} y = 2x^7 - \frac{1}{7x^7} - \sqrt[7]{2x};$$

$$\text{б)} y = \sqrt{x + \sqrt{x + \sqrt{x}}};$$

$$\text{в)} y = (3 - \sin^2 x)^3;$$

$$\text{г)} y = \frac{1 + \cos 2x}{x^3} + \sin(3x + 9);$$

$$\text{д)} y = e^{\sqrt{2x}} + 3;$$

$$2.7. \text{ а)} y = 3x^7 - \frac{1}{x^7} - \sqrt[7]{3x};$$

$$\text{б)} y = \sqrt{3 - 4x + 5x^2} + 4x \cdot \ln x;$$

$$\text{в)} y = \arcsin(3x^2 + 2);$$

$$\text{г)} y = \frac{\sin^2 x}{2 + \cos^2 x};$$

$$\text{д)} y = 3^{\sin^2 x};$$

$$2.8. \text{ а)} y = 4x^9 - \frac{4}{x^9} - \sqrt[9]{4x};$$

$$\text{б)} y = \sqrt[3]{1 + \sqrt{x + 3}}$$

$$\text{в)} y = \arctg \frac{2}{1 - x^2};$$

$$\text{г)} y = x \cdot \arccos \sqrt{4 - x^2};$$

$$\text{д)} y = 0,2^{\text{ctg}^2 x};$$

$$2.9. \text{ а)} y = 15x^3 - \frac{15}{x^3} + \sqrt[3]{x^2};$$

$$\text{з)} y = (x^2 + 2x + 2) \cdot e^{-x};$$

$$\text{и)} y = \sin(x + 6) - x \cdot \cos 4x;$$

$$\text{к)} y = (x^2)^{\frac{1}{x}}.$$

$$\text{е)} y = x \cdot \arctg 3x;$$

$$\text{ж)} y = \frac{9 - x^2}{9 + x^2};$$

$$\text{з)} y = 3 \sin^2 x \cdot \cos 2x;$$

$$\text{и)} y = e^{-x^2} + x^2 + \frac{3}{x};$$

$$\text{к)} y = x^{\arccos x}.$$

$$\text{е)} y = x\sqrt{1 + x^2}$$

$$\text{ж)} y = \frac{1 + e^x}{1 - e^x};$$

$$\text{з)} y = \sin^2 2x + \cos x;$$

$$\text{и)} y = \ln \text{tg} 5x;$$

$$\text{к)} y = (x + 1)^{2x}.$$

$$\text{е)} y = \arctg x^2 + 7x^6 + 2;$$

$$\text{ж)} y = \frac{x^2}{x^3 - 1};$$

$$\text{з)} y = x^2 \cdot \ln(x^2 + 1);$$

$$\text{и)} y = \frac{1}{\sqrt{x^2 + 1}} + 5\sqrt{x^3 + 1};$$

$$\text{к)} y = (\sin x)^{\text{tg} x}.$$

$$\text{е)} y = \frac{5 + \sin 5x}{4 - \cos 2x};$$

$$\text{ж)} y = (x^2 + 1) \cdot \arctg 4x;$$

$$\text{з)} y = (2x + 5) \cdot e^{-x^5};$$

$$\text{и)} y = \ln \sqrt{x + 1};$$

$$\text{к)} y = (\cos x)^{\sqrt{x}}.$$

$$\text{е)} y = e^x \cdot \cos x$$

$$\text{ж)} y = 3x^2 \cdot \ln x^3$$

$$\text{з)} y = \frac{3 + \sin 2x}{9 - e^{2x}};$$

$$\text{и)} y = (2x + 2 \cos x) \cdot e^{-x};$$

$$\text{к)} y = (\sin 2x)^{\cos x}$$

$$\text{е)} y = e^{\sin 4x + 8};$$

$$\text{б)} y = \sqrt{1 + \ln^2 x};$$

$$\text{в)} y = \frac{\cos 3x + 4}{\sin 3x - 4};$$

$$\text{г)} y = \operatorname{tg}^2 \sqrt{x+5} + 8x + 7;$$

$$\text{д)} y = (x + x^2)^x;$$

$$2.10. \text{а)} y = 5x^{10} + \frac{5}{x^{10}} + \sqrt[10]{5x};$$

$$\text{б)} y = \sqrt{1 + \cos^2 x^2};$$

$$\text{в)} y = x^2 \cdot \sqrt{1 - x^2};$$

$$\text{г)} y = \operatorname{arctg} \frac{x}{1 + \sqrt{1 - x^2}};$$

$$\text{д)} y = 5^{\sin x^3};$$

$$2.11. \text{а)} y = 3x^{11} + \frac{5}{x^{11}} + \sqrt[11]{x^3};$$

$$\text{б)} y = \left( \sin \frac{x}{2} - \cos \frac{x}{2} \right)^2;$$

$$\text{в)} y = \frac{x}{\sqrt{25 - x^2}};$$

$$\text{г)} y = \operatorname{arctg}(\ln x) + \ln(\sin x);$$

$$\text{д)} y = 2 \cdot \cos(4x + x^2);$$

$$2.12. \text{а)} y = 12x^7 - \frac{12}{x^7} + \sqrt[7]{x^2};$$

$$\text{б)} y = x^2 \cdot \arccos \frac{x}{2} - 4x;$$

$$\text{в)} y = \frac{x^5}{x^4 + 2};$$

$$\text{г)} y = \operatorname{arctg}^2 x + 6x^2;$$

$$\text{д)} y = 5^{\frac{1-x^2}{1+x^2}} + 7^{\cos 4x};$$

$$2.13. \text{а)} y = 6x^7 + \frac{7}{x^6} + \sqrt[6]{x^5};$$

$$\text{б)} y = \sqrt[3]{2 - x^2} \cdot \sqrt{x};$$

$$\text{в)} y = \frac{x^6}{6x^5 - 1};$$

$$\text{г)} y = \ln^3 \sin(3x + 3);$$

$$\text{д)} y = 2^{\operatorname{tg} 3x};$$

$$2.14. \text{а)} y = 12x^{14} + \frac{14}{x^2} - \sqrt[12]{x};$$

$$\text{б)} y = \sqrt[3]{x^2 + 3x};$$

$$\text{ж)} y = \frac{x}{x-1} - \ln 4x;$$

$$\text{з)} y = \frac{\arcsin x}{\sqrt{1-x^2}};$$

$$\text{и)} y = \cos^{100} x + \sin 100x;$$

$$\text{к)} y = 3^{\arccos 3x}.$$

$$\text{е)} y = \sin x \cdot \cos(7x + 5);$$

$$\text{ж)} y = (e^{\cos x} + 3)^2;$$

$$\text{з)} y = \ln \sin(3x + 5);$$

$$\text{и)} y = \frac{x^3}{x^2 - 1};$$

$$\text{к)} y = (x^3)^{\ln x};$$

$$\text{е)} y = (1 - x^2) \cdot \cos 2x;$$

$$\text{ж)} y = \sqrt[3]{x + x\sqrt{x}};$$

$$\text{з)} y = e^{-x} \cdot \sin 2x;$$

$$\text{и)} y = \ln^5(x^2 - 1);$$

$$\text{к)} y = (\sqrt{x})^{\frac{1}{x}}.$$

$$\text{е)} y = e^{\operatorname{ctg} 3x};$$

$$\text{ж)} y = \sqrt[4]{1 + \cos x^4};$$

$$\text{з)} y = \frac{4 + \cos 3x}{\sin(5x + 3)};$$

$$\text{и)} y = (x^3 + x^2) \cdot e^{-x};$$

$$\text{к)} y = x^{\arcsin^2 x}.$$

$$\text{е)} y = \ln(x^2 + 5);$$

$$\text{ж)} y = x^5 \cdot e^{-x};$$

$$\text{з)} y = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}};$$

$$\text{и)} y = \frac{\sin x + \cos x}{\sin x - \cos x};$$

$$\text{к)} y = (\sqrt{x})^{\sin x}.$$

$$\text{е)} y = 8x \cdot e^{-x^2};$$

$$\text{ж)} y = (3x + 1)^5 \cdot \cos 3x;$$

$$\text{B)} y = \frac{x}{x^2 + 2};$$

$$\text{Г)} y = \ln(2x^3 + 3x^2);$$

$$\text{Д)} y = 5^{\frac{x}{x+1}};$$

$$2.15. \text{a)} y = x^{15} + \frac{15}{x^2} - \sqrt{x};$$

$$\text{б)} y = (5x + x^3) \cdot \ln x^2;$$

$$\text{B)} y = \frac{x \cdot \cos x}{1 - \sin x} + 2 \sin 4x + 4;$$

$$\text{Г)} y = \arccos \frac{1}{2x^2};$$

$$\text{Д)} y = 0,7^{\arctg x};$$

$$2.16. \text{a)} y = 2x^5 + \frac{5}{x^2} - \sqrt[5]{2x};$$

$$\text{б)} y = \sqrt[3]{\cos^2 x + x^2};$$

$$\text{B)} y = \frac{x^3}{\sqrt{1+x^3}};$$

$$\text{Г)} y = x \cdot \arccos x - \sqrt{2-x^3};$$

$$\text{Д)} y = 7^{\ln^2 x};$$

$$2.17. \text{a)} y = x^7 + \frac{7}{x^3} - \sqrt[3]{7x};$$

$$\text{б)} y = \frac{\sin x}{1 + \ln \sin x};$$

$$\text{B)} y = (5 + x^3)^2 \cdot e^{-x};$$

$$\text{Г)} y = 2\sqrt{4x} - \frac{4}{\sqrt[3]{x^2+5}};$$

$$\text{Д)} y = 7^{\sqrt{\cos x}};$$

$$2.18. \text{a)} y = x^5 + \frac{5}{x^3} - \sqrt[3]{5x};$$

$$\text{б)} y = \frac{2 \sin 5x}{1 - \cos 3x};$$

$$\text{B)} y = \arcsin(\cos^2 x) + x^2;$$

$$\text{Г)} y = 2 \operatorname{tg}^3(x^3 + 2);$$

$$\text{Д)} y = 2^{\sin 3x};$$

$$2.19. \text{a)} y = 7x^2 + \frac{x^5}{5} - \sqrt[5]{x};$$

$$\text{б)} y = \ln \operatorname{ctg}^3 x;$$

$$\text{B)} y = \frac{x^7}{x^5 - 2};$$

$$\text{Г)} y = \arctg(\operatorname{tg}^2 x + 2);$$

$$\text{з)} y = \frac{\sin x}{3 \cos^2 x};$$

$$\text{и)} y = \operatorname{arctg}^2 e^x;$$

$$\text{к)} y = x^{\ln^2 x};$$

$$\text{е)} y = \cos(10x + x^3);$$

$$\text{ж)} y = (1 + \sqrt[3]{x})^3$$

$$\text{з)} y = \frac{7 - \cos 3x}{5 + \sin 5x};$$

$$\text{и)} y = \ln(4 + \sin 4x);$$

$$\text{к)} y = x^{\sin \sqrt{x}};$$

$$\text{е)} y = (3x + 2) \cdot \sin 3x;$$

$$\text{ж)} y = \ln^2 \operatorname{tg} 2x;$$

$$\text{з)} y = \frac{\cos x}{\sin^2 x} + \frac{\sin x}{\cos^2 x};$$

$$\text{и)} y = \arcsin(e^{7x});$$

$$\text{к)} y = (\sin 2x)^x;$$

$$\text{е)} y = e^x \cdot \sin 2x$$

$$\text{ж)} y = \operatorname{arctg} \sqrt{\frac{3-x}{x-2}};$$

$$\text{з)} y = \sqrt[3]{x + \sqrt[3]{x}};$$

$$\text{и)} y = \cos(3^x);$$

$$\text{к)} y = (\arcsin x)^{(x^2)};$$

$$\text{е)} y = (x^2 + 6) \cdot \ln 3x;$$

$$\text{ж)} y = \frac{x^2}{1-x} + \frac{9x+8}{x^3};$$

$$\text{з)} y = e^{3x} \cdot \cos 3x;$$

$$\text{и)} y = \operatorname{arctg}^2 \frac{1}{x};$$

$$\text{к)} y = (x+1)^{(x^2)};$$

$$\text{е)} y = \sin^2 6x + 3x^2;$$

$$\text{ж)} y = \sqrt{3x} \cdot \arcsin x^2;$$

$$\text{з)} y = \frac{1+e^{2x}}{1-e^{4x}};$$

$$\text{и)} y = \ln(x + \sqrt{x^2 + 2x + 3});$$

$$\text{д)} y = 2^{\frac{x}{1+x}} 7^{\cos 2x}$$

$$2.20. \text{а)} y = x^7 - \frac{x^6}{6} + \sqrt[6]{x};$$

$$\text{б)} y = \sqrt{x} - \operatorname{arctg} \sqrt{x};$$

$$\text{в)} y = 3x \cdot \sin^3 x - \cos^3 x;$$

$$\text{г)} y = \frac{\sqrt{x}}{\sqrt{x+1}};$$

$$\text{д)} y = \ln^2 \sin 3x;$$

$$2.21. \text{а)} y = x^5 - \frac{1}{x^5} + \sqrt[5]{x};$$

$$\text{б)} y = \sqrt{x+2\sqrt{x}};$$

$$\text{в)} y = \sqrt{x} \cdot e^{\sqrt{x}} + 5^{\cos 4x};$$

$$\text{г)} y = \operatorname{arctg}(7 \sin 3x);$$

$$\text{д)} y = \sqrt{x^2+1} + \sqrt{x^2-1};$$

$$2.22. \text{а)} y = \sqrt[4]{x^3} + \frac{5}{x^2} - \frac{3}{x^3} + 2;$$

$$\text{б)} y = \operatorname{tg}(x^2+3);$$

$$\text{в)} y = x^{\cos^2 x};$$

$$\text{г)} y = \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{x}{2}\right);$$

$$\text{д)} y = x^2 \cdot \arcsin(9x+2);$$

$$2.23. \text{а)} y = \sqrt[3]{x^2} - \frac{4}{x} - \frac{3}{x^2} + 3$$

$$\text{б)} y = \sqrt{2x - \sin 2x};$$

$$\text{в)} y = \sin^4 x + x^2 \cdot \cos^2 x;$$

$$\text{г)} y = \ln \sqrt{\frac{1+\sin x}{1-\sin x}};$$

$$\text{д)} y = 5^{\cos x^2};$$

$$2.24. \text{а)} y = \sqrt[3]{x^2} - \frac{3}{x^3} + \frac{2}{x^2} + 1;$$

$$\text{б)} y = \frac{1+\sin x}{1-\sin x};$$

$$\text{в)} y = \sqrt[3]{x} \cdot \operatorname{arctg} x;$$

$$\text{г)} y = (x^2 - x^3) \cdot e^{-x};$$

$$\text{д)} y = 15^{\ln^2 x};$$

$$\text{к)} y = (\sin \sqrt{x})^2.$$

$$\text{е)} y = 2 \ln \operatorname{ctg} \frac{x}{2}$$

$$\text{ж)} y = \frac{x}{6(x+1)};$$

$$\text{з)} y = \arcsin(e^{-4x});$$

$$\text{и)} y = 5^{\frac{1-x}{1+x}} + 3^{\cos 4x};$$

$$\text{к)} y = (3x)^{e^x}.$$

$$\text{е)} y = \frac{x^2}{1+x^2} + \frac{2+6x}{x^3};$$

$$\text{ж)} y = \ln^2 \operatorname{arctg} x;$$

$$\text{з)} y = \frac{\cos x}{\sin^2 x} + \ln\left(\operatorname{tg} \frac{x}{2}\right);$$

$$\text{и)} y = (\sqrt{x})^{\cos x};$$

$$\text{к)} y = \frac{x^2}{1+x^3}.$$

$$\text{е)} y = \frac{1+2\cos 3x}{1-\cos 2x};$$

$$\text{ж)} y = (0,9)^{\sqrt{x}};$$

$$\text{з)} y = \sin^3 \frac{x}{3} + \cos x;$$

$$\text{и)} y = 0,7^{(x^5)}$$

$$\text{к)} y = x\sqrt{x} \cdot (3 \ln x - 2).$$

$$\text{е)} y = e^{-x^2}$$

$$\text{ж)} y = 3 \operatorname{tg}^6 x + 7;$$

$$\text{з)} y = 4x \cdot \operatorname{arctg}(2x+9);$$

$$\text{и)} y = \frac{x^2}{(x-3)^2};$$

$$\text{к)} y = \left(\frac{1}{x}\right)^{\arccos x};$$

$$\text{е)} y = \operatorname{tg}(x^2 + \cos x);$$

$$\text{ж)} y = \sqrt{\frac{x}{2} - \sin \frac{x}{2}};$$

$$\text{з)} y = \frac{1}{2}(\sqrt{1-x^2} + \arcsin x);$$

$$\text{и)} y = 3x^3 + \ln^3 x;$$

$$\text{к)} y = (\sqrt{x})^{\operatorname{arctg} x};$$



$$2.25. \text{a)} y = 3x^5 - \frac{1}{x^5} + \frac{1}{x^4} + 2;$$

$$\text{б)} y = \operatorname{tg} x + \frac{2}{3} \operatorname{tg}^3 x + \frac{1}{5} \operatorname{tg}^5 x;$$

$$\text{в)} y = x^3 \cdot (x - 5 \cos x)^2;$$

$$\text{г)} y = \frac{x + \sqrt{x}}{x - 2\sqrt[3]{x}};$$

$$\text{д)} y = 5^{\sin 3x};$$

$$\text{е)} y = \sqrt{1+x^2} + 5^{\cos 3x};$$

$$\text{ж)} y = \ln^2 \sin x;$$

$$\text{з)} y = \arccos \frac{9-x^2}{9+x^2};$$

$$\text{и)} y = (1+9x) \cdot e^{-x^2};$$

$$\text{к)} y = (1+x)^{\cos x}.$$

$$2.26. \text{a)} y = 4x^2 - \frac{3}{2x^2} + \sqrt[3]{x};$$

$$\text{б)} y = \frac{2+e^{3x}}{9-e^{-4x}} \cdot x^2;$$

$$\text{в)} y = \operatorname{arctg}(x^2 + e^{3x});$$

$$\text{г)} y = \ln \operatorname{tg}(5x+1);$$

$$\text{д)} y = 3^{\ln 3x};$$

$$\text{е)} y = \ln(2x-3);$$

$$\text{ж)} y = \frac{3 + \sin 4x}{8 - \cos 3x};$$

$$\text{з)} y = (2x^3 + 5)^4 \cdot x^3;$$

$$\text{и)} y = \sin 5x + \cos 3x^3;$$

$$\text{к)} y = x^{\frac{2}{x}}.$$

$$2.27. \text{a)} y = 3x^5 - \frac{5}{x^5} + \sqrt[5]{5x+2};$$

$$\text{б)} y = \arcsin(3x^3 + 4);$$

$$\text{в)} y = (x+8) \cdot \operatorname{arctg} 4x^3;$$

$$\text{г)} y = \frac{2x^3 + 3x^2}{3x};$$

$$\text{д)} y = 4x \cdot (1 - 3 \ln x);$$

$$\text{е)} y = \frac{5x + \sin 4x}{\cos 2x - 4};$$

$$\text{ж)} y = \ln \cos(5x^3 + 4);$$

$$\text{з)} y = (\operatorname{ctg} 3x + 1)^5;$$

$$\text{и)} y = 5^{\sin 2x}$$

$$\text{к)} y = (\cos x)^x.$$

**Задание 3.** Проверить, удовлетворяет ли указанному уравнению данная функция  $u$ .

$$3.1. x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0, u = \frac{y}{x}$$

$$3.2. x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3(x^3 - y^3), u = \ln \frac{x}{y} + x^3 = y^3$$

$$3.3. \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u = \ln(x^2 + (y+1)^2)$$

$$3.4. y \frac{\partial^2 u}{\partial x \partial y} = (1 + y \ln x) \frac{\partial u}{\partial x}, u = x^u$$

$$3.5. x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u, u = \frac{xy}{x+y}$$

$$3.6. x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} = 0, u = e^{-xy}$$

$$3.7. a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}, u = \sin^2(x - ay)$$

$$3.8. x^2 \frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} = 0, u = y \sqrt{\frac{y}{x}}$$

$$3.9. \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0, u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

- 3.10.  $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$ ,  $u = e^{-\cos(x+ay)}$
- 3.11.  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ ,  $u = (x-y)(y-z)(z-x)$
- 3.12.  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u$ ,  $u = x \ln \frac{y}{x}$
- 3.13.  $y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} = 0$ ,  $u = \ln(x^2 + y^2)$
- 3.14.  $x^2 \frac{\partial u}{\partial x} - xy \frac{\partial u}{\partial y} + y^2 = 0$ ,  $u = \frac{y^2}{3x} + \arcsin(xy)$
- 3.15.  $x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy$ ,  $u = 0$ ,  $u = e^{-xy}$
- 3.16.  $\frac{\partial^2 u}{\partial x \partial y} = 0$ ,  $u = \operatorname{arccctg} \frac{x+y}{1-xy}$
- 3.17.  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ,  $u = \ln(x^2 + y^2 + 2x + 1)$
- 3.18.  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + u = 0$ ,  $u = \frac{2x+3y}{x^2+y^2}$
- 3.19.  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{2y}{u}$ ,  $u = \sqrt{2xy + y^2}$
- 3.20.  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$ ,  $u = (x^2 + y^2) \operatorname{tg} \frac{x}{y}$
- 3.21.  $9 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ,  $u = e^{-(x+3y)} \sin(x+3y)$
- 3.22.  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$ ,  $u = x e^{\frac{y}{x}}$
- 3.23.  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ,  $u = \operatorname{arctg} \frac{y}{x}$
- 3.24.  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ ,  $u = \operatorname{arctg} \frac{x}{y}$
- 3.25.  $\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x^2} = 0$ ,  $u = \ln(x + e^{-y})$
- 3.26.  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ ,  $u = \arcsin \frac{x}{x+y}$
- 3.27.  $\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} = \frac{u}{y^2}$ ,  $u = \frac{y}{(x^2 - y^2)^5}$