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Author(s): Gary M. Erickson
Source: *Management Science*, Vol. 38, No. 12 (Dec., 1992), pp. 1732-1749
Published by: INFORMS
Stable URL: http://www.jstor.org/stable/2632707
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MANAGEMENT SCIENCE
Vol. 38, No. 12, December 1992
Printed in U.S.A.

EMPIRICAL ANALYSIS OF CLOSED-LOOP DUOPOLY ADVERTISING STRATEGIES*

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Closed-loop (perfect) equilibria in a Lanchester duopoly differential game of advertising competition are used as the basis for empirical investigation. Two systems of simultaneous nonlinear equations are formed, one from a general Lanchester model and one from a constrained model. Two empirical applications are conducted. In one involving Coca-Cola and Pepsi-Cola, a formal statistical testing procedure is used to detect whether closed-loop equilibrium advertising strategies are used by the competitors rather than open-loop strategies. In the second application, involving Anheuser-Busch and Miller, the general model is estimated. Results indicate that closed-loop equilibria better explain dynamic advertising competition than do open-loop equilibria. Also, closed-loop equilibrium advertising strategies implied by model estimates show that competitive advertising levels may or may not be monotonic in market share.
(MARKETING—COMPETITIVE STRATEGY, ADVERTISING; GAMES—NONCOOPERATIVE, DIFFERENTIAL)

Introduction

Advertising is often considered, and rightly so, to have a competitive role to play in the ongoing struggle for market success. In markets that involve direct competition, a firm needs not only to develop its own effective advertising campaigns but also to be mindful of the advertising activities of its rivals. Empirical studies (e.g., Telser 1962; Little 1979; Carpenter et al. 1988) show that competitive advertising can have a negative impact on a firm's sales. As well, examples from the popular press serve to emphasize the importance of staying competitive in terms of advertising. Notable examples include the brewing industry, in which the two leading advertisers, Anheuser-Busch and Miller Brewing, "are threatening to run away with the beer business" (*Business Week* 1989), and the "cola war" involving Coca-Cola and Pepsi-Cola (Morris 1987).

Advertising competition involving direct market rivals is inherently dynamic. That is, an effective competitor will recognize the ongoing competitive challenges to its market position and act accordingly, a situation which requires dynamic adjustments to its advertising strategy in the attempt to maintain or improve its position. Theoretical and empirical study of advertising competition needs to recognize the dynamic nature of such competition.

The present study provides empirical investigation of dynamic advertising competition, a study that not only investigates the effectiveness of competitive advertising, but also how dynamically competitive advertising strategies are formed. A game theory setting is adopted, more specifically a differential game formulation involving the Lanchester model (Kimball 1957; Little 1979), in which duopolistic competitors adopt advertising strategies that change with market share. Two empirical studies are conducted. One involves the market share struggle between Coca-Cola and Pepsi-Cola, and uses a formal statistical procedure to investigate which kind of equilibrium advertising strategy is used, closed-loop or open-loop. The second study involves the two dominant brewing companies, Anheuser-Busch and Miller, and involves estimation of a general Lanchester model.

* Accepted by Jehoshua Eliashberg; received October 23, 1990. This paper has been with the author 3 months for 3 revisions.

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Previous Empirical Research on Advertising Competition

There have been numerous studies, beginning with the work by Telser (1962), that have linked share of advertising or relative advertising (to the total advertising of competitors) to market share.[1] An implication of such research is that an increase in a competitor's advertising is likely to have a negative effect on a firm's market share. The general conclusion that competitive advertising influences market share is not particularly an open question, although there may be managerial interest in specific applications.

A more open question is how competitors set their advertising levels, and what are the primary influences on advertising in a competitive setting. Various studies (e.g., Lambin 1970a, b, 1976; Grabowski and Mueller 1971; Wildt 1974; Lambin, Naert, and Bultez 1975; Metwally 1978; Hanssens 1980; Lancaster 1984) have investigated the possibility that competitors react to a rival's advertising, on a lagged basis. Generally, *positive* advertising reactions have been found, although Wildt (1974); Hanssens (1980); and Lancaster (1984) find limited reactions in terms of advertising. A complication arises from a study by Roberts and Samuelson (1988), who investigate whether firms anticipate competitive advertising reactions when they establish their own advertising goodwill levels. Roberts and Samuelson find that firms appear to determine their advertising goodwills as if they expect *negative*, not *positive*, competitive advertising reactions.

Certain studies have attempted to investigate the extent to which a competitor's advertising depends on its level of market share or sales. Such a relationship would be expected if the firm uses its advertising to maintain a desirable share or sales level, or if the firm sets its advertising budget on the basis of achieved sales levels. While some studies can detect no definite pattern (Wildt 1974; Lambin 1976; Lancaster 1984), the majority of studies find a significant effect of either contemporaneous or lagged market share or sales on advertising levels. Interestingly, some find *positive* effects (Lambin 1970a; Lambin, Naert, and Bultez 1975; Metwally 1978), while others find *negative* effects (Cowling et al. 1975; Brown 1978). Bass (1969) finds *both* in the same study.

Limited theoretical insight has been gained through empirical work to date with regard to the formation of competitive advertising strategies. The positive advertising reactions detected by certain studies clash with the Roberts and Samuelson (1988) finding that firms anticipate negative competitive reactions. The literature is also not consistent regarding the effect on advertising levels of a brand's sales level or market share.

Modeling frameworks in previous empirical research have been limited. In particular, the situation in which competitors act simultaneously in an information sense—each competitor makes its advertising decisions while inferring, but not observing, those of its rivals (Eliashberg and Chatterjee 1985)—has not been studied to any substantial degree. The approach taken in the present paper is that such inferrals are made based on observed market shares, that market shares influence the advertising decisions of the competitors in a closed-loop fashion. In dynamic settings, *differential games* (Case 1979) are often used to model advertising competition.[2] However, empirical investigation of advertising competition in differential game frameworks is at a very primitive stage; Chintagunta and Vilcassim (1991) provide an example of such an investigation.

Differential Games and Closed-Loop Advertising Strategies

Differential games are so named because they involve differential equations to define the change across time of variables of interest termed *state* variables. Sales, market share,

[1] See Erickson (1991, Chapter 1) for a review. Also see Hanssens, Parsons, and Schultz (1990, Chapter 6).

[2] The reader should note that differential games are not the only approach to modeling dynamic interactions. Alternatively, a supergame framework could be adopted (Friedman 1986), as could the Markovian approach introduced by Maskin and Tirole (1988).

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and advertising goodwill have been state variables assumed in differential games involving advertising.[3] The present study is concerned with duopolistic competition for market share. For a duopoly, define M to be the market share for competitor 1, with competitor 2's market share being $1 - M$. Further, define A_1 and A_2 to be the advertising rates of the two competitors. In a differential game setting, market share M and the advertising levels A_1, A_2, can vary across time, with M assumed to change according to some function of the advertising variables and current market share:

$$\dot{M} \equiv \frac{dM}{dt} = f(M, A_1, A_2), \qquad M(0) \text{ given.} \tag{1}$$

The advertising variables are under the control of the two competitors. It is assumed that the competitors advertise so as to maximize their discounted profits over an infinite time horizon:

$$i \max \int_0^\infty e^{-rt} h_i(M, A_1, A_2), \qquad i = 1, 2, \tag{2}$$

where the natural constraints $A_i \geq 0$ and $0 \leq M \leq 1$ must be satisfied. A differential game formulation is attractive because it encompasses the time element directly. Such a formulation captures the *dynamic* nature of advertising competition.

It is generally assumed that the competitors cannot cooperate in setting their advertising strategies. As such, *Nash equilibria* are sought. A Nash equilibrium is a pair of strategies, one for each competitor, which has the property that no competitor would like unilaterally to change its strategy (Moorthy 1985). In a Nash equilibrium, each strategy is a competitor's best strategy, given the strategies of its rival, where "best" means maximizing the profit integral in (2).

Some complications are involved in determining a Nash equilibrium for a differential game. There are two kinds of Nash equilibria that can be pursued: *open-loop*, in which advertising is a function of time only, $A_i = A_i(t, M(0))$, given a starting value for market share, and *closed-loop*, for which advertising is a function not only of time but also of the current state of the system which is summarized by the current value of the state variable, $A_i = A_i(t, M, M(0))$. Unfortunately, open-loop and closed-loop equilibria are generally different (Jorgensen 1982). By far the most frequently used approach in differential games has been to develop open-loop equilibria, primarily because they are easier to compute (Case 1979).

In open-loop strategies, the competitors commit at the outset to specific time paths of advertising expenditures. Open-loop equilibria are by definition *time consistent*, in that if at some intermediate point the competitors are asked to reconsider their strategies they would refuse to change them (Fershtman 1987a). However, if commitments are not feasible, and an open-loop equilibrium depends upon initial values of state variables, such an equilibrium is not *subgame perfect*, in that it does not necessarily constitute an equilibrium for every subgame that may start at a different point (Fershtman 1987a, b). The same reasoning holds for closed-loop equilibria that depend upon beginning values of the state variables. To be subgame perfect, an equilibrium must not depend upon initial conditions. Specifically, strategies $A_i(t, M)$ that depend upon current values of state variables as well as time and that do not depend upon initial conditions are termed *feedback* strategies (Fershtman 1987b).

The critical deficiency of open-loop strategies is precisely that, once determined, they are fixed; open-loop advertising levels may change across time, but the trajectory cannot be changed once the game has started. A marketing manager is not so likely to want to put advertising on such an automatic control; he/she would wish to monitor the market

[3] See Erickson (1991, Chapter 2) for a review of differential game models of advertising competition.

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situation as it proceeds across time and modify advertising, when needed, to correct the situation. Marketing managers need *closed-loop* equilibrium strategies.

In general, closed-loop equilibria have been difficult to obtain, since they tend to involve partial differential equations (Starr and Ho 1969; Fershtman 1987a). This would seem to deter development in the area of closed-loop equilibria until partial differential equation theory can be advanced beyond its present state. An approach by Case (1979), however, offers hope for a class of problems, those involving a single state variable, for which only ordinary, and not partial, differential equations are required. In particular, Case's approach can be applied to competitive situations involving duopolistic competition for market share.

Case (1979, pp. 210–215) considers what he terms *perfect equilibria*, which are time-invariant (stationary) functions of state variables.[4] In the present context, advertising strategies $A_i(M)$ vary with the market share state variable but not otherwise with time; such strategies are also not dependent on initial values of market share. Perfect equilibria are best viewed as representing *autonomous* situations, in which the market share and profit relationships do not depend explicitly on time (except through a discount factor applied to profits), e.g., models of market share rivalry in mature markets. Models involving growth or decline would qualify as autonomous models if the growth or decline were determined purely by the state and control variables in the problem. The assumption of time-invariance is not particularly limiting, since advertising can vary across time as the market share state variable does. Assume (1) and (2) to hold. Case formally shows the validity of the following procedure. Define the Hamiltonians

$$H_i = h_i(M, A_1, A_2) + k_i f(M, A_1, A_2), \qquad i = 1, 2, \tag{3}$$

where the k_i are *costate* variables that bring the dynamic market share constraint (1) into the maximization problem for each competitor; k_i can be considered a shadow price of market share. The Hamiltonians have a useful economic interpretation in that the last term on the right of each equation in (3) represents future profits resulting from a current change in market share. That is, derivation of optimal advertising through its effect on the Hamiltonians means that not only are current profit effects accounted for, but also are the effects on future profits that result from a current change in market share that is caused by advertising (k_i represents the change in discounted future profits for competitor i with respect to a change in market share M). Now determine $\hat{A}_1(M, k_1, k_2)$ and $\hat{A}_2(M, k_1, k_2)$ that form a Nash equilibrium for the auxiliary game

$$i \max H_i, \qquad i = 1, 2. \tag{4}$$

Define the *Hamilton-Jacobi* equations

$$\begin{aligned} &h_i(M, \hat{A}_1(M, V_1'(M), V_2'(M)), \hat{A}_2(M, V_1'(M), V_2'(M))) \\ &\quad + V_i'(M) f(M, \hat{A}_1(M, V_1'(M), V_2'(M)), \hat{A}_2(M, V_1'(M), V_2'(M))) \\ &\qquad = rV_i(M) + c_i, \qquad i = 1, 2, \end{aligned} \tag{5}$$

where the c_i are arbitrary real numbers. The functions $V_i(M)$ are called *value functions*, in that they represent the values to the competitors, in terms of discounted profit and with optimal advertising time paths, for different starting levels of M. The value functions are related to the costate variables k_i through the identity $k_i = V_i'(M)$. If the system of ordinary differential equations in (5) can be solved for $V_1(M)$ and $V_2(M)$, a perfect equilibrium is derived through the following relationships:

$$A_i^*(M) = \hat{A}_i(M, V_1'(M), V_2'(M)), \qquad i = 1, 2. \tag{6}$$

[4] This definition of "perfect" equilibria differs from what is elsewhere defined as "perfect" (e.g., Friedman 1986).

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An equilibrium (6), it should be noted, forms an optimal strategy for each competitor against the other, for any starting value of market share M. An interesting aspect of the perfect equilibria defined by Case is that they are defined in terms of parameters c_i. That is, Case shows that any solution of (5), for any real values of c_i, leads to a perfect equilibrium. This implies that there are a number, indeed an infinite number, of equilibrium advertising strategies that could be adopted by the competitors.

A Lanchester Model of Advertising Competition

The *Lanchester* model was introduced by Kimball (1957) and advanced by Little (1979) as a flexible modeling structure for analyzing advertising competition. The following version of the model, generalized to allow for advertising effects that are not strictly proportional, is analyzed in the present study:

$$\dot{M} = \beta_1 A_1^{\alpha_1}(1 - M) - \beta_2 A_2^{\alpha_2} M \tag{7}$$

where M is competitor 1's market share. The model interprets advertising as being used to capture market share from one's rival. The Lanchester model simply but elegantly captures the essence of dynamic competition.

Variations of the Lanchester model have been studied analytically by Case (1979); Erickson (1985, 1991); and Sorger (1989). Case (1979, pp. 215–219) develops perfect equilibria for a two-player advertising game. Erickson (1985) studies open-loop equilibria in steady state and in the transition to steady state. In Erickson (1991, Chapter 3), open-loop equilibria are compared to closed-loop equilibria (perfect equilibria à la Case). Sorger (1989) provides a qualitative analysis of open-loop and feedback equilibria in a model that combines Lanchester-type dynamics with "excess advertising" effects (i.e., those related to the simple difference between the advertising of the two competitors, $A_1 - A_2$). Each of these studies involves a duopoly, and none (except for Erickson 1985, with a brief example) offers empirical validation. Horsky (1977) and Nguyen (1987) provide empirical analysis as well as analytical development involving Lanchester-type models, but in a single-decision-maker framework.

Of particular note is a study by Chintagunta and Vilcassim (1991), in which estimation of a Lanchester model is used to develop closed-loop equilibria which are compared to open-loop equilibria. The differences between the present study and that by Chintagunta and Vilcassim are fundamental. The most important is that the present study views the relationships involving market share and the advertising of competitors as being simultaneously determined, through closed-loop equilibrium advertising strategies and the dynamic market share relationship (7); the closed-loop strategies are estimated *simultaneously* with the market share response model. Chintagunta and Vilcassim, on the other hand, estimate the market share response relationship with ordinary least squares regression and use the parameter estimates to simulate closed-loop strategies. This amounts to a difference in the maintained hypotheses in the two studies regarding the formation of advertising strategies. The present study assumes that advertising levels arise out of a closed-loop equilibrium (or, more appropriately, advertising levels are determined *as if* they arise from a closed-loop equilibrium). Another key difference between the two studies is that the present study allows for a broader set of closed-loop strategies.

The following specific profit objectives are assumed for the competitors:

$$i \max \int_0^\infty e^{-rt}(g_i M_i - A_i)\,dt, \qquad i = 1, 2, \tag{8}$$

where for notational convenience $M_1 = M$, $M_2 = 1 - M$, and the g_i are gross profit rates (in terms of market share). We wish to develop equilibria of the type defined by Case (1979), which we shall refer to as *closed-loop*. Details as to how this is done are found

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in the Appendix. Analytical solutions cannot be derived for the general case, but are available for a discount rate $r = 0$. Such solutions can be viewed as good approximations for small values of r (Case 1979, pp. 217–218).[5] The following system of relationships result:

$$g_i M_i + \frac{1-\alpha_i}{\alpha_i} A_i - \frac{\beta_{3-i}}{\alpha_i \beta_i} A_i^{1-\alpha_i} A_{3-i}^{\alpha_{3-i}} \frac{M_i}{M_{3-i}} = c_i, \qquad i = 1, 2, \tag{9}$$

a system in which the advertising variables A_i are defined implicitly in terms of market share M.

Explicit expressions of advertising as functions of market share can be obtained with a special case of the general Lanchester model (7) in which $\alpha_i = 0.5$ for each competitor:

$$\dot{M} = \beta_1 \sqrt{A_1}(1-M) - \beta_2 \sqrt{A_2} M. \tag{10}$$

Define

$$C_i \equiv \frac{M_i}{M_{3-i}}, \qquad D_i \equiv \frac{\beta_i}{\beta_{3-i}}, \qquad E_i \equiv g_i M_i - c_i. \tag{11}$$

The relationships (9) lead to, after manipulation:

$$A_i(M) = \frac{2\dfrac{C_i^2}{D_i^2} E_{3-i} - E_i + 2\sqrt{E_i^2 - \dfrac{C_i^2}{D_i^2} E_i E_{3-i} + \dfrac{C_i^4}{D_i^4} E_{3-i}^2}}{3}, \qquad i = 1, 2. \tag{12}$$

It needs to be emphasized that the closed-loop (perfect) solutions in (9) and (12) *are not unique*. Since the solutions depend on the constants c_i, which can take on any real value, there are an infinite number of closed-loop advertising strategies that each competitor could adopt.[6] Which particular closed-loop strategies are actually adopted by the competitors can be viewed as an *empirical* question; the c_i are parameters to be estimated in an econometric analysis.

A way of interpreting the c_i constants conceptually is to note that, at steady state (when $\dot{M} = 0$), it can be shown that the following relationships hold:

$$c_i = g_i M_i - A_i, \qquad i = 1, 2. \tag{13}$$

That is, at steady state, each firm's constant is equal to the net profit that firm is receiving. A way of viewing the c_i values, therefore, is that they represent the profit the firms expect to make once the competitive situation reaches steady state.

Empirical Applications

The system of relationships (7), (9) from the general Lanchester model, and (10), (12) from the constrained model, become the basis for empirical analysis. Note that the nature of closed-loop advertising strategies implies simultaneity between advertising and market share. The dynamical equation (7) (or (10)) states that market shares are influenced by advertising efforts. Assuming closed-loop advertising strategies means that the reverse is also true; advertising levels are determined by market shares. This means that empirical analysis should consider the evolution of market shares and advertising levels

[5] Strictly, a positive discount rate is needed due to the infinite time horizon assumed.

[6] Erickson (1991, Chapter 3) offers analytical and numerical analysis of closed-loop equilibria for the Lanchester duopoly model. A conclusion that develops is that there is a wide variety of possible closed-loop patterns in the way advertising depends on market share. For example, the relationship can be a monotonic one, but non-monotonic patterns are also likely.

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as a *simultaneous system*, especially if the data are available only on an annual basis, such as is the case in the applications in the present paper.

A further consideration is that information is shared across equations, in that parameters to be estimated appear in more than one equation. As such, *full-information* methods should be used to estimate the system (Theil 1971, p. 528). In addition, relationships are nonlinear; maximum likelihood has been recommended for efficient estimation of nonlinear systems (Berndt et al. 1974).[7]

It is not suggested that the closed-loop model necessarily portrays actual decision-making behavior on the part of the competing firms. The assumption is, rather, that the model can be paramorphic in its description of competitive and dynamic advertising patterns, that is, that the actual advertising decisions can be viewed *as if* they were derived through such a model. The value of such an approach is to allow science to contribute to knowledge, to allow a productive way to combine theoretical and empirical analysis in the pursuit of understanding of competition in dynamic markets.

Two empirical analyses of closed-loop advertising strategies follow. The first involves the soft drink brands, Coca-Cola and Pepsi-Cola, and compares with a formal statistical testing procedure closed-loop strategies to open-loop strategies. The constrained Lanchester model (10) is assumed for the soft drink application due to requirements of the statistical procedures used that advertising be expressed explicitly in terms of market share (12). The empirical evidence indicates that closed-loop strategies are preferred. The second application involves the two brewing companies, Anheuser-Busch and Miller, and analyzes the general Lanchester model (7), (9) with full information maximum likelihood estimation (FIML in TSP, which allows implicit expressions such as those in (9)). The empirical analysis shows that the dynamic advertising patterns of the two competitors can be explained by a particular closed-loop (perfect) equilibrium.

Coca-Cola Versus Pepsi-Cola: Comparison of Closed-Loop and Open-Loop Strategies

Coca-Cola and Pepsi-Cola have been involved in a market share struggle for a number of years. At one point, Coca-Cola was the dominant brand in the market, but a challenge to Coke's leadership was mounted by Pepsi in the 1970s (*Business Week* 1983). Until the Coca-Cola Company introduced "new Coke" in 1985, Coke and Pepsi were by far the two largest selling soft drink brands. Advertising has been a major weapon for both brands in the "Cola War" (Morris 1987).

Advertising and market share data for the two brands are available from various issues of *Advertising Age*. Current advertising values are used in the analysis, since the decision problem (8) and the derived advertising relationships to be estimated (9), (12) are interpreted in current-dollar terms. The study involves data from 1968 through 1984 (the year before the "new Coke" introduction). Figure 1 shows advertising expenditures for the two brands during this period, and Figure 2 shows Coke's generally declining share of the combined sales of the two brands.

An intent of the present empirical analysis is to determine whether the two soft drink competitors use closed-loop or open-loop equilibrium advertising strategies, that is, whether the competitors adjust their advertising to changes in market share, or, alternatively, whether they have fixed their strategies to allow for changes in advertising across time but not due to market share developments. For the present analysis, the square-root form of the Lanchester model (10) is assumed, and Coca-Cola is considered to be competitor 1.

A statistical test of closed-loop versus open-loop equilibrium strategies involves the comparison of nonnested alternatives. A variety of tests have been developed for this type of situation. Due to nonlinearity, the approach used in the empirical application is

[7] An alternative procedure to full information maximum likelihood is nonlinear three-stage least squares.

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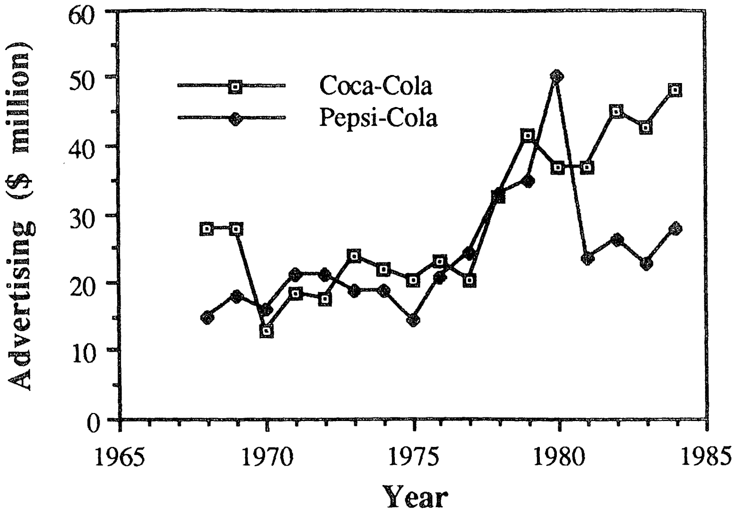

FIGURE 1. Coca-Cola and Pepsi-Cola Advertising.

the P test developed by Davidson and MacKinnon (1981), a nonlinear extension of their J test, which operates on the individual equations of the system. MacKinnon, White and Davidson (1983) show the validity of such tests for a variety of situations, including the presence of lagged dependent variables and when instrumental variable estimation is used to estimate the alternative models.

The Davidson and MacKinnon (1981) P test calls for, after the estimation of each alternative model, an additional regression that has been adjusted for the fitted values from what is considered the null hypothesis. Use the following general notation:

$$H_0\colon y = f(X, \beta),$$
$$H_1\colon y = g(Z, \gamma), \tag{14}$$

where H_0 is the null hypothesis, y is the dependent variable, X and Z represent sets of independent variables, and β and γ are parameter sets. The first step in the process is to

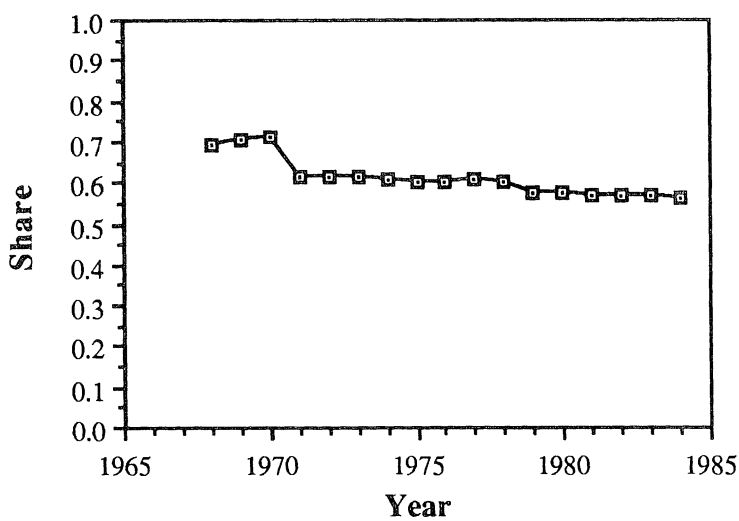

FIGURE 2. Coca-Cola's Share.

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estimate each hypothesis and obtain fitted values $\hat{f}$ and $\hat{g}$. The second step is to estimate the following model:

$$y-\hat{f}=\alpha(\hat{g}-\hat{f})+\hat{F}b \tag{15}$$

where $\hat{F}$ is a vector of derivatives of f with respect to β, evaluated at $\hat{\beta}$, and b is a vector of regression coefficients. Under H_0, the t statistic on the estimate $\hat{\alpha}$ is asymptotically $N(0, 1)$.

A statistical comparison of closed-loop equilibrium strategies to open-loop strategies requires that the alternative models be well specified, so that they can be estimated. For open-loop strategies, the following differential equations can be derived from necessary conditions:

$$\dot{A}_i = 2A_i\left(r+\frac{\beta_{3-i}\sqrt{A_{3-i}}}{M_{3-i}}-\frac{g_i\beta_i M_{3-i}}{2\sqrt{A_i}}\right), \qquad i=1, 2. \tag{16}$$

The open-loop strategies can be further specified through numerical solution of (16), given values for the parameters β_1, β_2, g_1, g_2, and r. Two-stage least squares (instrumental variable) estimation of (10) can be used to obtain consistent estimates of the advertising effectiveness parameters β_1 and β_2.[8] Initial estimation indicates the statistical equivalence of the two parameters and, in the interest of parsimony, equivalence is assumed for further estimation, which produces $\hat{\beta}_1=\hat{\beta}_2=0.0119$ (with a t statistic of 2.18).[9]

The parameters g_1, g_2, and r are unknown, but this can be accounted for. Numerical investigation shows two types of effects on open-loop strategies as these parameter values are varied: (1) shifting of the time paths, up or down, (2) squeezing toward or expansion away from zero. As such, the best fitting open-loop strategies can be captured in the empirical analysis by allowing for (and estimating) a linear combination of base case strategies. The particular base strategies used have β_1 and β_2 as estimated by two-stage least squares, $g_1=g_2=500$, and $r=0$.[10] Call the time-varying advertising paths from the base strategies OL_1 and OL_2, for Coca-Cola and Pepsi-Cola, respectively. The open-loop advertising equations to be estimated then become

$$A_1=w_{10}+w_{11}OL_1, \qquad A_2=w_{20}+w_{21}OL_2. \tag{17}$$

Estimated open-loop relationships (17) are compared to the closed-loop relationships estimated from (12).

The Davidson and MacKinnon (1981) P test is applied for each brand separately. Also, the test is applied for each hypothesis, open-loop and closed-loop, against each other. (The advertising equations in the closed-loop model are estimated with nonlinear two-stage least squares, and ordinary least squares is used for the open-loop model.) This results in the t values shown in Table 1, where the first hypothesis in each column heading is considered the null hypothesis.

Only the open-loop versus closed-loop comparison for Coca-Cola provides a significant t value. That is, for Coca-Cola, the open-loop hypothesis can be rejected in favor of the closed-loop hypothesis. At the same time, the closed-loop hypothesis cannot be rejected in favor of the open-loop model. Quite clearly, for Coca-Cola the closed-loop hypothesis is the preferred one. The results are more equivocal for Pepsi-Cola; the statistical evidence

[8] Two-stage least squares is used to avoid bias in the event that a simultaneous relationship exists between advertising and market share. Of course, two-stage least squares provides consistent estimates even if advertising is free of any influence of market share.

[9] Instruments used are lagged market share, a time variable (expressed as the number of years from the beginning of the data), and the time variable squared.

[10] It does not matter which base strategies are used. Different bases tried led to the same results in the P tests comparing open-loop and closed-loop solutions.

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TABLE 1

P Tests for Coca-Cola and Pepsi-Cola

	Open-Loop vs. Closed-Loop	Closed-Loop vs. Open-Loop
Coca-Cola	6.29	0.20
Pepsi-Cola	−1.22	0.21

does not allow accepting one hypothesis or the other for the Pepsi brand.[11] This also means that the closed-loop model cannot be rejected for Pepsi-Cola, and as a game-theoretic system of relationships involving Coke and Pepsi the evidence points toward a closed-loop equilibrium as providing a better explanation.

Estimation of the full closed-loop model reveals the parameter estimates listed in Table 2. Full information maximum likelihood (FIML in TSP) is used to take advantage of the information shared across the equations. It is interesting to note in particular that, according to the estimates of the parameters g_1 and g_2, the Coca-Cola brand appears to be more profitable on a gross profit basis than Pepsi-Cola. This perhaps reflects scale economies advantages had by the market leader.

Implied closed-loop advertising strategies for Coca-Cola and Pepsi-Cola, developed from the parameter estimates in Table 2, are shown in Figure 3. Both strategies exhibit increasing advertising amounts in response to a decline in the brand's own share. The increases become especially sharp near the share value of 0.6. As Coca-Cola's share drops below that value, advertising spending for Coca-Cola increases significantly; above 0.6, Pepsi's advertising rises sharply. In addition, it can be shown as an implication from the estimated model that a steady state—a situation in which market share would be expected to remain stable—exists at a share value of 0.590 (to three decimal places). This leads to the conclusion that, once Coca-Cola's share reaches 0.59, the share should stabilize. Variation from a share of 0.59 would trigger advertising patterns that would tend to bring the share back to that level.

Anheuser-Busch Versus Miller: Analysis of a General Lanchester Model

By being the only brewers able to maintain consistent growth and market success, Anheuser-Busch and Miller have made the national market for beer into essentially a struggle between two strong competitors (*Business Week* 1989).[12] This competitive sit-

TABLE 2

Closed-Loop Model Estimates for Coca-Cola and Pepsi-Cola

Parameter	Estimate	Standard Error	t
g_1	795	113	7.04
g_2	366	77	4.75
c_1	440	68	6.52
c_2	136	32	4.29

[11] Other evidence leads to the same conclusions. Comparing the sums of squared residuals (SSR) from various regressions provides conflicting results for Pepsi-Cola. For example, the SSR from the OLS regression of the open-loop equation is lower than that for the TSLS estimation of the closed-loop relationship. On the other hand, in FIML estimations of the full systems, the closed-loop SSR is better than the open-loop for Pepsi-Cola. For Coca-Cola, on the other hand, the closed-loop relationship always dominates the open-loop specification in terms of SSR, regardless of the estimation method.

[12] Together, Anheuser-Busch and Miller accounted for 62.4% of the total industry sales in 1988.

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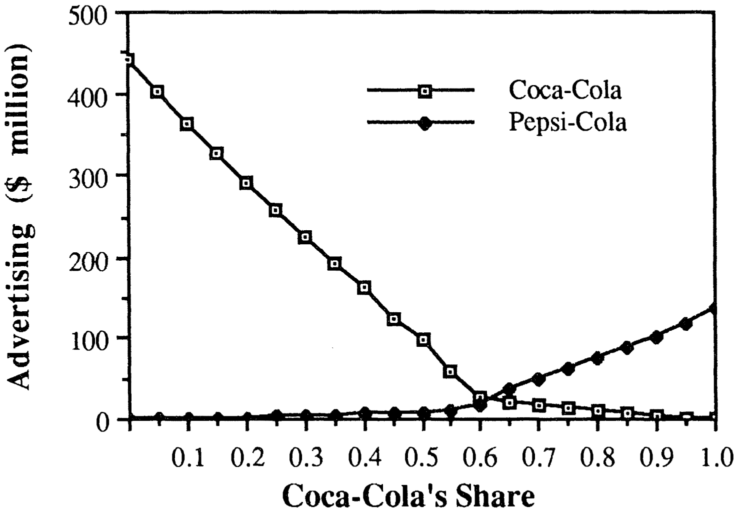

FIGURE 3. Closed-Loop Strategies for Coca-Cola and Pepsi-Cola.

uation was instigated in the early 1970s, when Philip Morris Inc. acquired Miller and began a campaign to increase the brewer's market position. Although not right away, Anheuser-Busch eventually "declared all-out war on Miller" (*Business Week* 1982, p. 52), and began escalating its own advertising expenditures. Two decades later, Anheuser-Busch retains the larger share of the market, but Miller (and only Miller) has provided a strong challenge. Other competitors have faded substantially and have not been able to mount a serious challenge to the two larger brewers. As a consequence, the rivalry between Anheuser-Busch and Miller in the beer industry can be viewed as an example of duopolistic competition.

Sales and advertising data for Anheuser-Busch and Miller[13] are available from various issues of *Advertising Age*. Figure 4 shows the dollar advertising expenditures of the two companies for the years 1971 through 1988, and Figure 5 shows Anheuser-Busch's share of the combined sales of the two brewers in the same period.[14] Even though Miller was not the second largest brewer in the market until 1977, the year 1971 was chosen as the beginning year for the data, since this was the first full year under Philip Morris's ownership, when the strategy for Miller underwent a basic change. Advertising for both competitors tended to increase over the period. In particular, Anheuser-Busch appeared to have sharply accelerated its advertising efforts in the late 1970s, after experiencing a sharp decline in share at the expense of Miller. Anheuser-Busch eventually halted the decline in its share, and was able to increase its share vis-a-vis Miller in the 1980s.

The primary intent of the present empirical application is to estimate the system of relationships (7), (9) developed from the general Lanchester model. Note that this allows not only estimation of parameters of advertising effectiveness (α_i, β_i) but also economic parameters internal to the competing firms—the gross profit values g_i, as well as the c_i constants which determine the particular closed-loop advertising strategies used by the competitors.

[13] Other brewing companies are not included in the analysis in order to focus on the rivalry between the two market leaders. As a conjecture, including other companies in the empirical analysis, if one could, would most likely result in nonsignificant estimated advertising effects for those companies, since they have not been effective in gaining share from the two major brewers, or even maintaining their own. Due caution is advised, however, since Miller did not have the second largest market share in the industry during the early part of the data period, 1971–1976.

[14] It should be noted that there is the potential for a data interval bias, since the available data interval of a year may not correspond to decision intervals.

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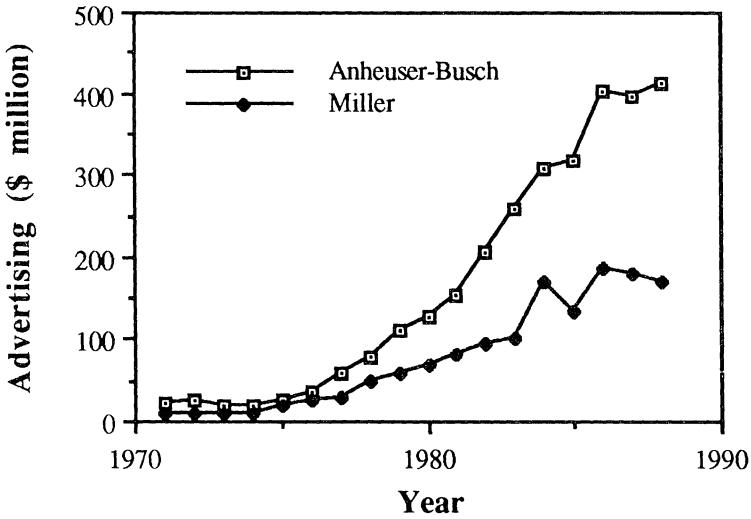

FIGURE 4. Advertising Expenditures for Anheuser-Busch and Miller.

FIML is used to estimate the system (7), (9), and Anheuser-Busch is considered to be competitor 1. Additional parameter constraints are needed for successful estimation; without further constraints, the estimation procedure does not converge and multicollinearity is a problem. The parameter estimates shown in Table 3 are obtained by setting the advertising elasticities of the two competitors equal to each other: $\alpha_1 = \alpha_2 = \alpha$.[15] Note that this still allows the advertising effectiveness of the competitors to differ through the parameters β_1 and β_2.

One result to note in Table 3 is that the advertising elasticity for the two brewers is estimated to be fairly low, 0.05102. As is shown by the estimates of β_1 and β_2, though,

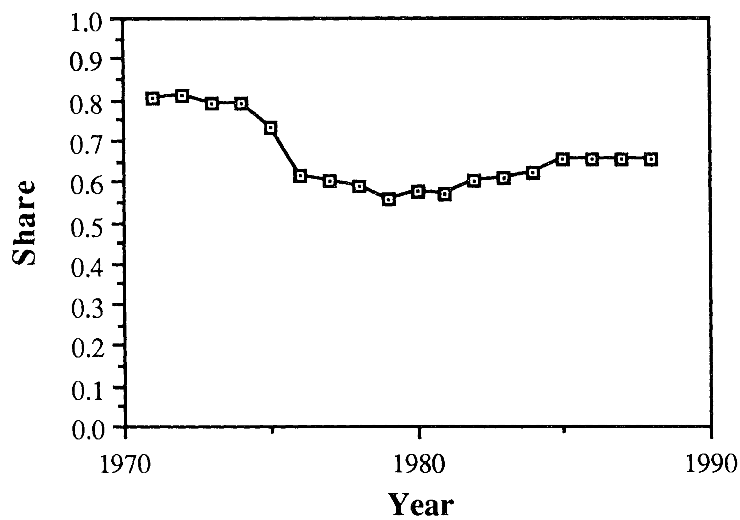

FIGURE 5. Anheuser-Busch's Share.

[15] Various regressions are consistent in supporting this constraint on the two elasticities. For example, nonlinear least squares regression of the market share relationship results in a difference in the estimates of the α_i that is not statistically significant.

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TABLE 3

Closed-Loop Model Estimates for Anheuser-Busch and Miller

Parameter	Estimate	Standard Error	t
α	0.05102	0.01616	3.16
β_1	0.08633	0.04769	1.81
β_2	0.04221	0.02338	1.81
g_1	6250	1905	3.28
g_2	4826	1167	4.13
c_1	4551	1350	3.37
c_2	1116	342	3.26

Anheuser-Busch appears to be more effective at advertising than Miller. It is interesting to compare these estimates with those from a more traditional analysis that simply estimates the market share response to the advertising of the two competitors. Nonlinear least squares estimation of the market share relationship (7) with[16] $\alpha_1 = \alpha_2 = \alpha$ finds the estimate of α to be 0.5990 (with a standard error of 0.2497), which is much greater than that shown in the table. In addition, the estimates of β_1 and β_2 at 0.01554 (standard error 0.01835) and 0.01462 (0.01454) are much lower than the FIML estimates shown in the table. It would appear that not considering market share and competitive advertising as a system of relationships can lead to a distorted view of advertising effects on market share.

Estimates of the g_1 and g_2 parameters in Table 3 show a difference between the two competitors in terms of gross contribution from market share.[17] The parameter values can be interpreted as indicating that Anheuser-Busch stands to gain $62.5 million in gross profit with each share point ($6.25 billion if they were to have the whole market). Miller, on the other hand, makes a smaller gross profit, $48.26 million with each share point. This difference in profitability is consistent with other reported evidence that Anheuser-Busch had higher unit profitability than Miller during the period studied (*Business Week* 1982), and could be due to economies of scale had by the larger brewer. Alternatively, the difference in gross contribution could be due to a difference in product mix. Finally, with the interpretation of the c_1 and c_2 parameters from (13), Anheuser-Busch apparently expects to achieve about four times Miller's profits, net of advertising, should the market achieve steady state.

Implied closed-loop equilibrium strategies for the competing brewers, in terms of differing levels of the Anheuser-Busch share, can be derived from (9) using Newton's method. The derived strategies are shown in Figure 6, and show an intriguing nonmonotonicity. In an interval around 0.7, the relationship for Anheuser-Busch, in particular, calls for extremely high, "off the chart," advertising levels. This apparently strange result occurs due to the attempt to construct a continuous curve over the entire domain of possible share values when only a limited set of such values are available for empirical analysis. Actually, the data include two distinct share intervals, [0.557, 0.656] and [0.731, 0.810], with the bulk of the data (13 points) in the former interval. There are no share values in the region around 0.7, and only one in the broad interval (0.656, 0.791), which encompasses the "spikes" in the closed-loop curves shown in Figure 6. For the share

[16] Estimation without this constraint shows a statistically insignificant difference in the elasticity estimates.

[17] The model and analysis assume that the gross profit rates are constant across time. What the impact would be of having nonconstant values for these parameters is not completely clear. One would think, however, that increasing profit margins would encourage increased advertising, the argument being that a larger profit margin means a greater profit impact from advertising. Conversely, one would expect decreased advertising from a declining profit margin.

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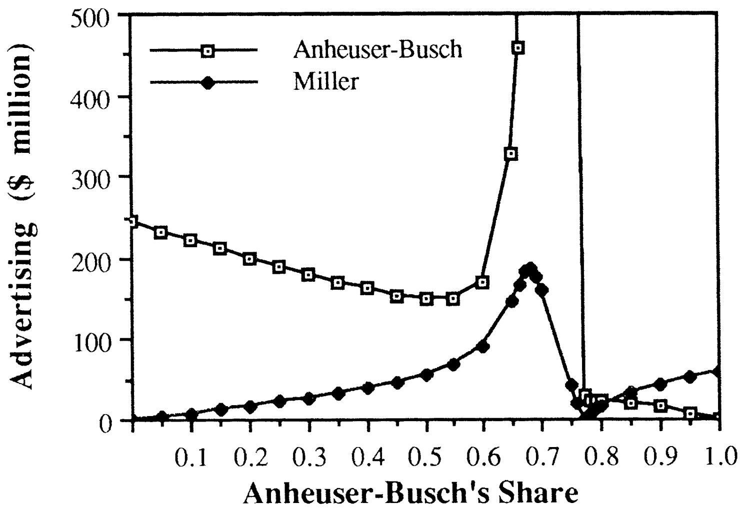

FIGURE 6. Closed-Loop Strategies for Anheuser-Busch and Miller.

intervals in which the two brewers have been operating, though, the implied strategies offer interesting insights. In particular, the nonmonotonistic patterns of the closed-loop strategies indicate that both brewers would increase advertising should Anheuser-Busch's share rise in the interval from 0.55 to 0.66. This perhaps reflects a combination of Miller's attempt to keep Anheuser-Busch's share in the lower interval with Anheuser-Busch's desire to return to its dominance in the days before Miller's challenge, when its share was in the 0.8 area; Miller increases its advertising as Anheuser-Busch threatens to return to its previous domination of the market, and Anheuser-Busch also increases it advertising in the attempt to dominate. The strategy curves in Figure 6 indicate that both brewers would advertise at a lower level if Anheuser-Busch were to be successful in establishing its former dominance.

Figures 7 and 8 show how advertising levels for the closed-loop equilibrium strategies would have proceeded over the data period, and also compares these to the actual advertising amounts. The bad fit near 1975 is due to the "spike" problem noted in Figure

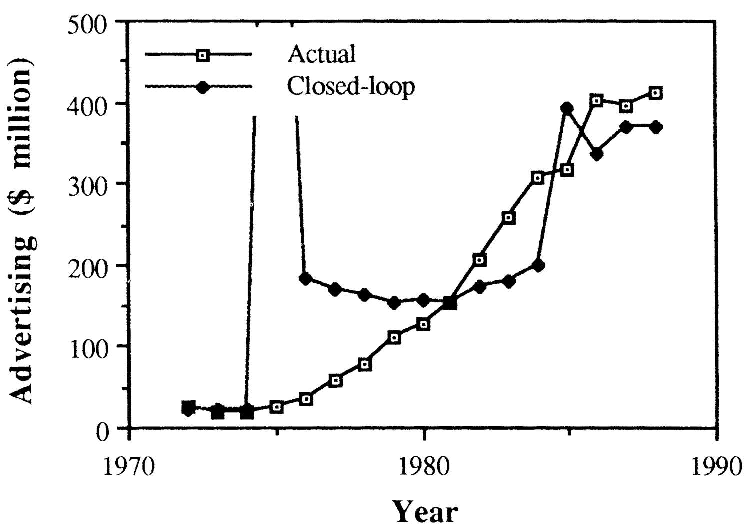

FIGURE 7. Closed-Loop and Actual Advertising for Anheuser-Busch.

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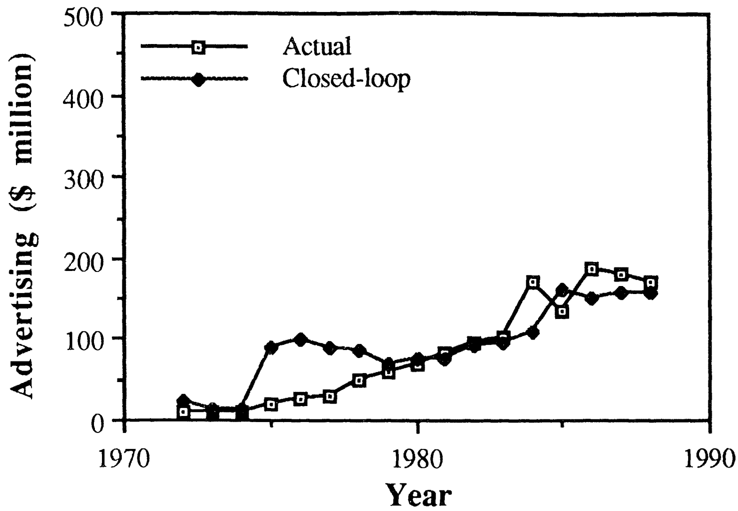

FIGURE 8. Closed-Loop and Actual Advertising for Miller.

6; in 1975, the Anheuser-Busch share was 0.731, the lone data point in the interval (0.656, 0.791) in which the spikes appear in the strategy curves for the two brewers. In the latter part of the period, though, the closed-loop strategies are able to pick up the growth in advertising by both competitors.

Perhaps a better way to view the performance of the closed-loop equilibrium advertising strategies is to observe how well they are able to predict beyond the data. Subsequent to the initial gathering of the data, the 1989 advertising figures became available; in 1989, Anheuser-Busch spent $388 million on advertising (rounded to the nearest million) and Miller $150 million. Closed-loop equilibrium advertising strategies would have called for spending levels of $357 million and $154 million, respectively. By comparison, a naive model, based on the best-fitting (over the 1971–1988 data period) proportion of previous year's sales, would have called for $230 million for Anheuser-Busch and $106 million for Miller.[18] At least for a one-period prediction, the closed-loop model appears to be perform well.

Discussion

Empirical analysis of closed-loop advertising strategies is advanced in the present study as an insightful way to study advertising competition. It would appear that closed-loop strategies better capture the dynamic evolution of advertising expenditures than do open-loop strategies. This is indicated through a formal statistical testing procedure in the Coca-Cola versus Pepsi-Cola application. That closed-loop strategies outperform open-loop strategies also agrees with the conclusion in Chintagunta and Vilcassim (1991). Furthermore, in each empirical application in the present study, the dynamic advertising patterns of the competitors involved can be explained (with error) by a particular closed-loop (perfect) equilibrium.[19]

[18] Other possible candidates for alternative models could be used. For example, Bass (1969) estimates sales-to-advertising relationships for filter and nonfilter cigarette types with a log linear model.

[19] In the Anheuser-Busch versus Miller case, a pattern observed is of generally increasing advertising on the part of both competitors. An alternative explanation for this pattern could come from market growth (Erickson 1985). There has been some growth in the beer industry, but it has been moderate. The combined sales of Anheuser-Busch and Miller have increased about 8% a year, on average. The overall beer industry has grown very slowly, at about 3% a year. Advertising for the two brewers, on the other hand, has increased at an average rate of 18% a year.

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An intriguing finding that arises from the empirical applications is that market share effects on advertising in closed-loop equilibrium strategies may not be monotonic; they are in the Coca-Cola versus Pepsi-Cola situation, but not in that involving Anheuser-Busch and Miller. This may well explain the mixed results, some negative and some positive market share effects on advertising levels, found in previous research. Also, the Anheuser-Busch versus Miller application indicates that there are situations in which closed-loop equilibrium strategies call for a positive interaction between the advertising of two competitors. The Coca-Cola versus Pepsi-Cola situation, on the other hand, suggests the opposite, that if one competitor is increasing its advertising, the other should be decreasing its. While we need to be careful to point out that the present study is not concerned with competitive advertising *reactions*, the detection of both positive and negative advertising interactions provides an explanation for the ambiguity arising from previous research on competitive advertising reactions.

Conclusions

There are certainly limitations to the study that should be noted. A duopoly and a single state variable are assumed. As well, a zero discount rate assumption is needed to derive strategies as approximations for those under small values of the discount rate. In addition, even though a dynamic modeling framework is used, certain aspects of the model are assumed to be fixed—total demand, gross profit rates. Also, the stationary nature of the closed-loop equilibria studied does not allow examination of more general closed-loop strategies, that may vary in time as well as in terms of market share. Relaxation of these restrictions would make it very difficult, if not impossible, to develop a model that can be analyzed empirically. It may be that we are limited to studying competitive situations that match or approximate the restrictions assumed.

The contributions of the present study should also be recognized. The use of the perfect equilibrium concept allows the derivation of a system of equations that can be empirically analyzed with full information maximum likelihood methods; restricting the equilibrium strategies to be time invariant still leaves an abundance of possible strategies. It is indicated that closed-loop (perfect) equilibrium advertising strategies are to be preferred to open-loop strategies. The empirical applications show that valuable insights into advertising competition can be obtained through econometric analysis of competitive closed-loop advertising strategies.

Appendix

The Hamiltonians become

$$H_i = g_i M_i - A_i + k_i(\beta_1 A_1^{\alpha_1} M_2 - \beta_2 A_2^{\alpha_2} M_1). \tag{A1}$$

Setting $\partial H_i/\partial A_i = 0$ yields

$$\hat{A}_i(M, k_1, k_2) = ([-1]^{i+1}\alpha_i\beta_i k_i M_{3-i})^{1/(1-\alpha_i)}. \tag{A2}$$

(For nonnegative $\hat{A}_i$ we must have k_1 nonnegative and k_2 nonpositive.) Identifying value functions according to

$$V_i'(M) = k_i \tag{A3}$$

the Hamilton-Jacobi equations are as follows, where $V_i' = V_i'(M)$, $V_i = V_i(M)$,

$$g_i M_i - ([-1]^{i+1}\alpha_i\beta_i V_i' M_{3-i})^{1/(1-\alpha_i)} + V_i'(\beta_1[\alpha_1\beta_1 V_1' M_2]^{\alpha_1/(1-\alpha_1)} M_2 - \beta_2[-\alpha_2\beta_2 V_2' M_1]^{\alpha_2/(1-\alpha_2)} M_1) = rV_i + c_i. \tag{A4}$$

An equivalent system can be defined in terms of the advertising variables, rather than the value functions. From (A2) and (A3),

$$V_i' = (-1)^{i+1}\frac{A_i^{1-\alpha_i}}{\alpha_i\beta_i M_{3-i}} \quad \text{and} \tag{A5}$$

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$$V_i = (-1)^{i+1} \int \frac{A_i^{1-\alpha_i}\, dM}{\alpha_i \beta_i M_{3-i}}. \tag{A6}$$

Substituting (A5) and (A6) into (A4) yields

$$g_i M_i + \frac{1-\alpha_i}{\alpha_i} A_i - \frac{\beta_{3-i}}{\alpha_i \beta_i} A_i^{1-\alpha_i} A_{3-i}^{\alpha_{3-i}} \frac{M_i}{M_{3-i}} = (-1)^{1+i} r \int \frac{A_i^{1-\alpha_i}\, dM}{\alpha_i \beta_i M_{3-i}} + c_i. \tag{A7}$$

Setting $r = 0$ in (A7) yields the relationships in (9).

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