

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{3} =$$

$$= \frac{5}{6} + \sum_{n=1}^{\infty} \frac{6}{n^2 \pi^2} \left((-1)^n + 1 - 2 \cos \frac{n\pi}{3} \right) \cos \frac{n\pi x}{3}.$$

Это разложение справедливо в области непрерывности данной функции.

ЗАДАЧИ ДЛЯ КОНТРОЛЬНЫХ ЗАДАНИЙ

КОНТРОЛЬНАЯ РАБОТА № 4

I. Найти неопределенные интегралы. Правильность полученных результатов проверить дифференцированием.

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| 1. 1) $\int \frac{x dx}{7 + x^2};$ | 2) $\int \frac{(x + 18) dx}{x^2 - 4x - 12};$ | 3) $\int (3 - x) \cos x dx;$ |
| 2. 1) $\int \frac{dx}{\sin^2 \frac{x}{5}};$ | 2) $\int \frac{(x + 4) dx}{x^2 - 2x - 8};$ | 3) $\int x \ln(1 - 3x) dx;$ |
| 3. 1) $\int \frac{dx}{\sqrt{5 - x^2}};$ | 2) $\int \frac{(x + 23) dx}{x^2 + x - 20};$ | 3) $\int x e^{-7x} dx;$ |
| 4. 1) $\int \frac{dx}{5x + 3};$ | 2) $\int \frac{(x + 12) dx}{x^2 - x - 6};$ | 3) $\int \operatorname{arctg} 4x dx;$ |
| 5. 1) $\int \sin(2 - 3x) dx;$ | 2) $\int \frac{(x + 19) dx}{x^2 - 2x - 15};$ | 3) $\int \sqrt{x^3} \ln x dx;$ |
| 6. 1) $\int e^{\frac{1}{4}x-2} dx;$ | 2) $\int \frac{(5x + 6) dx}{x^2 + 4x - 12};$ | 3) $\int x \sin 5x dx;$ |
| 7. 1) $\int \frac{dx}{7 + 4x^2};$ | 2) $\int \frac{(5x - 7) dx}{x^2 - x - 20};$ | 3) $\int (2x + 5) \sin x dx;$ |
| 8. 1) $\int \frac{dx}{\cos^2 2x};$ | 2) $\int \frac{5x dx}{x^2 + x - 6};$ | 3) $\int \frac{\ln x dx}{\sqrt{x}};$ |
| 9. 1) $\int \cos\left(\frac{x}{3} - 4\right) dx;$ | 2) $\int \frac{(5x + 2) dx}{x^2 + 2x - 8};$ | 3) $\int \arcsin \frac{x}{3} dx;$ |

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| 10. 1) $\int \frac{dx}{\sqrt[3]{(2x+1)}};$ | 2) $\int \frac{(5x+1)dx}{x^2+2x-15};$ | 3) $\int xe^{3x}dx;$ |
| 11. 1) $\int \frac{e^x dx}{\sqrt[3]{1-e^x}};$ | 2) $\int \frac{19-4x}{2x^2+x-3}dx;$ | 3) $\int (5x-2)\ln xdx;$ |
| 12. 1) $\int x\sqrt{3-x^2}dx;$ | 2) $\int \frac{2x+9}{x^2+5x+6}dx;$ | 3) $\int x\cos^2 2xdx;$ |
| 13. 1) $\int \frac{\arctg x dx}{1+x^2};$ | 2) $\int \frac{x+9}{x^2+2x-3}dx;$ | 3) $\int \ln(3+x^2)dx;$ |
| 14. 1) $\int \sin 2x\sqrt{2-\cos^2 x}dx;$ | 2) $\int \frac{2x+27}{x^2-x-12}dx;$ | 3) $\int x\arcsin xdx;$ |
| 15. 1) $\int \frac{\sin x dx}{1-\cos x};$ | 2) $\int \frac{4x+31}{2x^2+11x+12}dx;$ | 3) $\int (2-x)\sin xdx;$ |
| 16. 1) $\int \frac{\sqrt[3]{\ln x} dx}{x};$ | 2) $\int \frac{11x-2}{x^2+x-2}dx;$ | 3) $\int (1-\ln x)dx;$ |
| 17. 1) $\int \frac{1-\tg x}{\cos^2 x}dx;$ | 2) $\int \frac{17-2x}{x^2-5x+4}dx;$ | 3) $\int (3x+4)\cos xdx;$ |
| 18. 1) $\int \frac{x^2}{8+x^3}dx;$ | 2) $\int \frac{9-2x}{x^2-5x+6}dx;$ | 3) $\int \arctg(4x)dx;$ |
| 19. 1) $\int \frac{\sin 2x}{\cos^2 x+3}dx;$ | 2) $\int \frac{4x+27}{2x^2-x-6}dx;$ | 3) $\int x\ln^2 xdx;$ |
| 20. 1) $\int \frac{x^2 dx}{\cos^2 x^3};$ | 2) $\int \frac{x-13}{x^2-2x-8}dx;$ | 3) $\int x^2 \sin 3xdx;$ |

II. Вычислить определенный интеграл.

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| 1. $\int_2^7 \frac{\sqrt{x+2}dx}{x}$ | 2. $\int_{-\frac{3}{4}}^0 \frac{3xdx}{\sqrt{(x+1)^3}}$ |
| 3. $\int_0^1 \frac{\sqrt{x}dx}{4-x}$ | 4. $\int_{-8}^0 \frac{dx}{5-\sqrt[3]{x^2}}$ |
| 5. $\int_2^7 \frac{dx}{\sqrt{x-3}}$ | 6. $\int_{-4}^1 \frac{xdx}{\sqrt{(5-x)^3}}$ |

$$7. \int_{-3/4}^0 \frac{dx}{2 - \sqrt{1+x}}$$

$$9. \int_{-1/4}^0 \frac{dx}{1 + \sqrt{3x+1}}$$

$$11. \int_0^4 \frac{\sqrt{x} dx}{4+x}$$

$$13. \int_0^3 \frac{x^2 + \sqrt{1+x}}{\sqrt{1+x}} dx$$

$$15. \int_{25}^{49} \frac{\sqrt{x} dx}{x-6}$$

$$17. \int_{-8}^0 \frac{3\sqrt{x^2}}{\sqrt[3]{x^2} + 3} dx$$

$$19. \int_1^2 \frac{dx}{2 + \sqrt[4]{x-1}}$$

$$8. \int_{-1}^1 \frac{dx}{8 + \sqrt[3]{x^2}}$$

$$10. \int_{-1}^0 \frac{dx}{4 + \sqrt[3]{x^2}}$$

$$12. \int_3^6 \frac{\sqrt{x-3}}{x} dx$$

$$14. \int_2^1 \frac{\sqrt{x-2}}{1 + \sqrt{x-2}} dx$$

$$16. \int_0^1 \frac{x dx}{\sqrt{2x+7}}$$

$$18. \int_4^9 \frac{dx}{\sqrt{x}(x-1)}$$

$$20. \int_0^4 \frac{dx}{\sqrt{x+5}}$$

III. Вычислить несобственный интеграл или установить его расходимость.

$$1. \int_{-3}^5 \frac{dx}{\sqrt[3]{x+3}}$$

$$3. \int_e^\infty \frac{dx}{x \ln^2 x}$$

$$5. \int_1^3 \frac{x dx}{x-1}$$

$$7. \int_e^{\pi/4} \frac{\cos x dx}{\sin^2 x}$$

$$9. \int_1^3 \frac{dx}{\sqrt{(x+1)^3}}$$

$$11. \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$2. \int_0^\infty x e^{-x^2} dx$$

$$4. \int_4^5 \frac{dx}{(x-4)^2}$$

$$6. \int_{-\pi/2}^\infty \operatorname{tg} x dx$$

$$8. \int_0^\infty \frac{dx}{x^2 + 2x + 5}$$

$$10. \int_0^{\pi/2} \operatorname{ctg} x dx$$

$$12. \int_0^\infty \frac{dx}{9x^2 + 1}$$

$$13. \int_0^8 \frac{3x+2}{\sqrt[3]{x}} dx$$

$$15. \int_0^8 \frac{dx}{x \ln^3 x}$$

$$17. \int_{-1}^0 \frac{dx}{\sqrt{(x+1)^3}}$$

$$19. \int_{-5}^{-4} \frac{dx}{\sqrt[3]{(x+5)^4}}$$

$$14. \int_0^{\infty} e^{-5x} dx$$

$$16. \int_{-\infty}^0 e^{-x^2} x dx$$

$$18. \int_0^{\infty} \frac{dx}{4x^2 + 1}$$

$$20. \int_{14}^{\infty} \frac{dx}{\sqrt[4]{x+2}}$$

IV. Вычислить площадь плоской фигуры, ограниченной заданными кривыми. Сделать чертеж области.

$$1. 3x^2 - 4y = 0, \quad 2x - 4y + 1 = 0$$

$$2. 3x^2 + 4y = 0, \quad 2x - 4y - 1 = 0$$

$$3. 2x + 3y^2 = 0, \quad 2x + 2y + 1 = 0$$

$$4. 3x^2 - 4y = 0, \quad 2x + 4y - 1 = 0$$

$$5. 3x^2 + 4y = 0, \quad 2x + 4y + 1 = 0$$

$$6. 2x - 3y^2 = 0, \quad 2x + 2y - 1 = 0$$

$$7. 3x^2 - 2y = 0, \quad 2x - 2y + 1 = 0$$

$$8. 4x + 3y^2 = 0, \quad 4x + 2y + 1 = 0$$

$$9. 3x^2 - 2y = 0, \quad 2x + 2y - 1 = 0$$

$$10. 4x - 3y^2 = 0, \quad 4x + 2y - 1 = 0$$

$$11. y = x^3 + 3, \quad x = 0, \quad y = x - 1, \quad x = 2$$

$$12. y = x^3 + 2, \quad x = 0, \quad y = x - 2, \quad x = 2$$

$$13. y = x^3 + 1, \quad x = 0, \quad y = x - 3, \quad x = 2$$

$$14. y = x^3 - 1, \quad x = 0, \quad y = x - 5, \quad x = 2$$

$$15. y = x^3 - 2, \quad x = 0, \quad y = x - 6, \quad x = 2$$

$$16. y = x^3 + 3, \quad x = 0, \quad y = x + 7, \quad x = -2$$

$$17. y = x^3 + 2, \quad x = 0, \quad y = x + 6, \quad x = -2$$

$$18. y = x^3 + 1, \quad x = 0, \quad y = x + 5, \quad x = -2$$

19. $y = x^3 - 1, \quad x = 0, \quad y = x + 3, \quad x = -2$

20. $y = x^3 - 2, \quad x = 0, \quad y = x + 2, \quad x = -3$

V. Вычислить объем тела, образованного вращением вокруг оси Ox кривой L .

1. $x^2 - y = 0, \quad x = -1, \quad y = 0$

2. $x^2 + y = 0, \quad x = 0, \quad y = -1$

3. $x^2 + 2y = 0, \quad x = 1, \quad y = 0$

4. $x^2 - y = 0, \quad x = 0, \quad y = 1$

5. $x^2 - y = 0, \quad x = 1, \quad y = 0$

6. $x - y^2 = 0, \quad x = 1, \quad y = 0$

7. $x - y^2 = 0, \quad x = 0, \quad y = -1$

8. $x + y^2 = 0, \quad x = -1, \quad y = 0$

9. $x - y^2 = 0, \quad x = 0, \quad y = 1$

10. $x + y^2 = 0, \quad x = 0, \quad y = 1$

11. $y = -4x^3, \quad x = 0, \quad y = 4$

12. $y = -4x^3, \quad x = 1, \quad y = 0$

13. $y = 4x^3, \quad x = 0, \quad y = 4$

14. $y = 4x^3, \quad x = 1, \quad y = 0$

15. $y = 1 + 8x^3, \quad x = 0, \quad y = 9$

16. $y = 4x^3, \quad x = 0, \quad y = -4$

17. $y = -4x^3, \quad x = -1, \quad y = 0$

18. $y = -4x^3, \quad x = 0, \quad y = -4$

19. $y = 4x^3, \quad x = -1, \quad y = 0$

20. $y = 1 + 8x^3, \quad x = -1/2, \quad y = 1$

КОНТРОЛЬНАЯ РАБОТА № 5

I. Изменить порядок интегрирования в двойном интеграле. Сделать чертеж области интегрирования.

1. $\int_{-1}^0 dx \int_{-8x^2}^{-2x+6} f(x, y) dy$

2. $\int_0^1 dy \int_{-4y-4}^{-8y^3} f(x, y) dy$

3. $\int_0^1 dx \int_{8x^3}^{4x+4} f(x, y) dy$

4. $\int_{-1}^0 dy \int_{2y-6}^{8y^3} f(x, y) dy$

5. $\int_{-1}^0 dx \int_{4x-4}^{8x^3} f(x, y) dy$

6. $\int_0^1 dy \int_{8y^3}^{2y+6} f(x, y) dy$

7. $\int_0^1 dx \int_{-2x-6}^{-8x^3} f(x, y) dy$

8. $\int_{-1}^0 dy \int_{-8y^3}^{-4y+4} f(x, y) dy$