

-1	-	-14 7 -7	-3 3 -3	-	-	-	-17	17
0	-	-	-2 2 0	0 50 0	2 2 0	-	0	0
1	-	-	-1 1 1	0 10 10	6 6 6	-	5	5
2	-	-	-	0 4 8	7 7 14	6 3 6	13	26
$V = \sum n_{uv} v$	-4	-13	-2	18	20	6		$\sum vU = 72$
uV	12	26	2	0	20	12	$\sum uV = 72$	\uparrow \leftarrow Контроль

Далее находим h_1 и h_2 , \bar{x} и \bar{y} , σ_x и σ_y :

$$h_1 = 9 - 4 = 5; h_2 = 20 - 10 = 10;$$

$$\bar{x} = \bar{u} \cdot h_1 + C_1 = -0,11 \cdot 5 + 19 = 18,45;$$

$$\bar{y} = \bar{v} \cdot h_2 + C_2 = 0,25 \cdot 10 + 30 = 32,5; \quad \sigma_x = h_1 \cdot \sigma_u = 5 \cdot 0,95 = 4,75;$$

Подставив найденные величины в формулу (6), получим

$$\bar{y}_x - 32,5 = 0,82 \cdot \frac{9,7}{4,75} (x - 18,45), \quad \text{или} \quad \bar{y}_x = 1,67x + 2,2.$$

ЗАДАЧИ ДЛЯ КОНТРОЛЬНЫХ ЗАДАНИЙ

КОНТРОЛЬНАЯ РАБОТА № 7

I. Найти общее решение дифференциального уравнения и частное решение, удовлетворяющее начальному условию $y = y_0$ при $x = x_0$.

1. $y' \sin x - y \cos x = 1; y_0 = 0, x_0 = \frac{\pi}{2}.$
2. $y' - y \sin x = e^{-\cos x} \sin 2x; y_0 = 3, x_0 = \frac{\pi}{2}.$
3. $y' + \frac{2y}{x} = -x^2; y_0 = 1, x_0 = 3.$
4. $y' + y = \frac{e^{-x}}{1+x^2}; y_0 = 2, x_0 = 0.$
5. $(1+x^2)y' - 2xy = (1+x^2)^2; y_0 = 5, x_0 = -2.$
6. $xy' - 2y = x^3 \cos x; y_0 = 1, x_0 = \pi.$
7. $y'x \ln x - y = 3x^3 \ln^2 x; y_0 = 0, x_0 = e.$
8. $y' + 2xy = xe^{-x^2}; y_0 = 4, x_0 = 0.$
9. $y' \cos x - 2y \sin x = 2; y_0 = 3, x_0 = 0.$
10. $y' - \frac{3y}{x} = x^3 e^x; y_0 = e, x_0 = 1.$
11. $xy' - 3y = x^4 e^x; y_0 = e, x_0 = 1.$
12. $y' \cos x + y \sin x = 1; y_0 = 2, x_0 = 0.$
13. $y' + \frac{y}{x} = \frac{\sin x}{x}; y_0 = 1, x_0 = \frac{\pi}{2}.$
14. $y' - \frac{y}{x} = -2 \ln x; y_0 = 1, x_0 = 1.$
15. $xy' + 2y = \frac{1}{x}; y_0 = 1, x_0 = 3.$
16. $y' - y \cos x = -\cos x; y_0 = 3, x_0 = 0.$
17. $y' + 2xy = e^{-x^3} \sin x; y_0 = 1, x_0 = 0.$
18. $x^2 y' + xy + 1 = 0; y_0 = 2, x_0 = 1.$
19. $y' - y \operatorname{tg} x = \frac{1}{\cos x}; y_0 = 5, x_0 = 0.$
20. $y' - \frac{2y}{x+1} = (x+1)^3; y_0 = \frac{1}{2}, x_0 = 0.$

III. Найти общее решение дифференциального уравнения и частное решение,

удовлетворяющее начальным условиям.

1. $y''x \ln x = y'$; $y(e) = e - 1$, $y'(e) = 1$.
2. $y''' \cos^4 x = -\sin 2x$; $y(\pi) = 0$, $y'(\pi) = 2$, $y''(\pi) = -1$.
3. $2xy'' = y'$; $y(9) = 8$, $y'(9) = 3$.
4. $y'' = y' \ln y'$; $y(0) = 0$, $y'(0) = 1$.
5. $y''(x^2 + 1) = 2xy'$; $y(0) = 1$, $y'(0) = 3$.
6. $y'' \cos x + y' \sin x = 0$; $y(0) = -\frac{1}{4}$, $y'(0) = 2$.
7. $y'' + y' = e^{-x}$; $y(0) = 1$, $y'(0) = 1$.
8. $y'' - 6y' + 9y = 9x^2 - 12x + 2$; $y(0) = 1$, $y'(0) = 3$.
9. $y'' + 9y = 36e^{3x}$; $y(0) = 0$, $y'(0) = 0$.
10. $y'' + 2y' - 8y = 3 \sin x$; $y(0) = -1$, $y'(0) = -\frac{3}{2}$.
11. $y'' + 6y' + 13y = 8e^{-x}$; $y(0) = \frac{2}{3}$, $y'(0) = 2$.
12. $y'' - 4y' + 8y = 8x^2 + 4$; $y(0) = 2$, $y'(0) = 3$.
13. $y'' + y' - 5y = 50 \cos x$; $y(0) = 3$, $y'(0) = 5$.
14. $y'' + 2y' + 5y = 13e^{2x}$; $y(0) = 1$, $y'(0) = 4$.
15. $y'' - 4y' + 5y = 10x$; $y(0) = 10$, $y'(0) = 6$.
16. $y'' - 4y' + 4y = 3x - x^2$; $y(0) = 3$, $y'(0) = \frac{4}{3}$.
17. $y'' - 6y' + 9y = 4e^x$; $y(0) = 3$, $y'(0) = 8$.
18. $y'' - 4y' + 4y = -169 \sin 3x$; $y(0) = -12$, $y'(0) = 16$.
19. $y'' + 2y' - 8y = 16x + 4$; $y(0) = 2$, $y'(0) = 6$.
20. $y'' - 4y' + 5y = 5x^2 - 4$; $y(0) = \frac{2}{25}$, $y'(0) = \frac{3}{5}$.

IV. Найти общее решение системы дифференциальных уравнений.

1.
$$\begin{cases} \frac{dx_1}{dt} = 12x_1 + 5x_2 \\ \frac{dx_2}{dt} = 5x_1 + 12x_2 \end{cases}$$
2.
$$\begin{cases} \frac{dx_1}{dt} = x_1 + 3x_2 \\ \frac{dx_2}{dt} = x_1 - x_2 \end{cases}$$

$$3. \begin{cases} \frac{dx_1}{dt} = x_1 + 4x_2 \\ \frac{dx_2}{dt} = x_1 + x_2 \end{cases}$$

$$5. \begin{cases} \frac{dx_1}{dt} = x_1 - x_2 \\ \frac{dx_2}{dt} = x_2 - x_1 \end{cases}$$

$$7. \begin{cases} \frac{dx_1}{dt} = x_1 + x_2 \\ \frac{dx_2}{dt} = 4x_2 - 2x_1 \end{cases}$$

$$9. \begin{cases} \frac{dx_1}{dt} = x_1 - 5x_2 \\ \frac{dx_2}{dt} = -2x_1 - 2x_2 \end{cases}$$

$$11. \begin{cases} \frac{dx_1}{dt} = x_1 + 3x_2 \\ \frac{dx_2}{dt} = 2x_1 \end{cases}$$

$$13. \begin{cases} \frac{dx_1}{dt} = -3x_1 + 2x_2 \\ \frac{dx_2}{dt} = 5x_1 - 6x_2 \end{cases}$$

$$15. \begin{cases} \frac{dx_1}{dt} = 4x_1 - x_2 \\ \frac{dx_2}{dt} = -2x_1 + 5x_2 \end{cases}$$

$$17. \begin{cases} \frac{dx_1}{dt} = x_1 + 2x_2 \\ \frac{dx_2}{dt} = 3x_1 - 4x_2 \end{cases}$$

$$4. \begin{cases} \frac{dx_1}{dt} = 3x_1 + x_2 \\ \frac{dx_2}{dt} = -4x_1 - 2x_2 \end{cases}$$

$$6. \begin{cases} \frac{dx_1}{dt} = 2x_1 - 4x_2 \\ \frac{dx_2}{dt} = x_1 - 3x_2 \end{cases}$$

$$8. \begin{cases} \frac{dx_1}{dt} = 4x_1 + 5x_2 \\ \frac{dx_2}{dt} = x_1 \end{cases}$$

$$10. \begin{cases} \frac{dx_1}{dt} = 2x_1 + 6x_2 \\ \frac{dx_2}{dt} = 3x_1 - x_2 \end{cases}$$

$$12. \begin{cases} \frac{dx_1}{dt} = 4x_1 - x_2 \\ \frac{dx_2}{dt} = -2x_1 + 3x_2 \end{cases}$$

$$14. \begin{cases} \frac{dx_1}{dt} = x_1 - 2x_2 \\ \frac{dx_2}{dt} = -3x_1 - 4x_2 \end{cases}$$

$$16. \begin{cases} \frac{dx_1}{dt} = -x_1 + 3x_2 \\ \frac{dx_2}{dt} = 2x_1 - 2x_2 \end{cases}$$

$$18. \begin{cases} \frac{dx_1}{dt} = -3x_1 + 2x_2 \\ \frac{dx_2}{dt} = 5x_1 \end{cases}$$

$$19. \begin{cases} \frac{dx_1}{dt} = x_1 - 2x_2 \\ \frac{dx_2}{dt} = -3x_1 + 6x_2 \end{cases}$$

$$20. \begin{cases} \frac{dx_1}{dt} = x_1 + 2x_2 \\ \frac{dx_2}{dt} = 3x_1 + 2x_2 \end{cases}$$

V. Решить уравнение колебания струны методом Фурье.

$$1. \varphi(x) = \begin{cases} \frac{2hx}{3}; 0 \leq x \leq \frac{3}{2}, \\ 2h(2-x); \frac{3}{2} \leq x \leq 2; \end{cases} \quad \psi(x) = x(2-x).$$

$$2. \varphi(x) = \begin{cases} \frac{-2hx}{3}; 0 \leq x \leq \frac{3}{2}, \\ \frac{2h(x-3)}{3}; \frac{3}{2} \leq x \leq 2; \end{cases} \quad \psi(x) = x(3-x).$$

$$3. \varphi(x) = \begin{cases} hx; 0 \leq x \leq 1, \\ h; 1 \leq x \leq 2, \\ h(3-x); 2 \leq x \leq 3; \end{cases} \quad \psi(x) = x(3-x).$$

$$4. \varphi(x) = \begin{cases} 2hx; 0 \leq x \leq \frac{1}{2}, \\ \frac{2h(4-x)}{3}; \frac{1}{2} \leq x \leq 4; \end{cases} \quad \psi(x) = x(4-x).$$

$$5. \varphi(x) = \begin{cases} -4hx; 0 \leq x \leq \frac{1}{4}, \\ \frac{4h(x-5)}{19}; \frac{1}{4} \leq x \leq 5; \end{cases} \quad \psi(x) = x(5-x).$$

$$6. \varphi(x) = \begin{cases} -hx; 0 \leq x \leq 1, \\ -h; 1 \leq x \leq 3, \\ h(x-4); 3 \leq x \leq 4; \end{cases} \quad \psi(x) = x(4-x).$$

$$7. \varphi(x) = \begin{cases} \frac{hx}{3}; 0 \leq x \leq 3, \\ \frac{h(6-x)}{3}; 3 \leq x \leq 6; \end{cases} \quad \psi(x) = x(6-x).$$

$$8. \quad \varphi(x) = \begin{cases} \frac{-2hx}{7}; 0 \leq x \leq \frac{7}{2}, \\ \frac{2h(x-7)}{7}; \frac{7}{2} \leq x \leq 7; \end{cases} \quad \psi(x) = x(7-x).$$

$$9. \quad \varphi(x) = \begin{cases} \frac{hx}{2}; 0 \leq x \leq 2, \\ h; 2 \leq x \leq 4, \\ h(5-x); 4 \leq x \leq 5; \end{cases} \quad \psi(x) = x(5-x).$$

$$10. \quad \varphi(x) = \begin{cases} \frac{-hx}{3}; 0 \leq x \leq 3, \\ -h; 3 \leq x \leq 6, \\ h(x-7); 6 \leq x \leq 7; \end{cases} \quad \psi(x) = x(7-x).$$

$$11. \quad \varphi(x) = \begin{cases} \frac{2h}{5}x; 0 \leq x \leq \frac{5}{2}, \\ \frac{2h}{5}(5-x); \frac{5}{2} \leq x \leq 5; \end{cases} \quad \psi(x) = \begin{cases} 2x, 0 \leq x \leq \frac{5}{2} \\ 2(5-x), \frac{5}{2} \leq x \leq 5 \end{cases}$$

$$12. \quad \varphi(x) = \begin{cases} \frac{2h}{3}x; 0 \leq x \leq \frac{3}{2}, \\ \frac{2h}{3}(3-x); \frac{3}{2} \leq x \leq 3; \end{cases} \quad \psi(x) = \begin{cases} 2x, 0 \leq x \leq \frac{3}{2} \\ 2(3-x), \frac{3}{2} \leq x \leq 3 \end{cases}$$

$$13. \quad \varphi(x) = \begin{cases} \frac{2h}{7}x; 0 \leq x \leq \frac{7}{2}, \\ \frac{2h}{7}(7-x); \frac{7}{2} \leq x \leq 7; \end{cases} \quad \psi(x) = \begin{cases} 4x, 0 \leq x \leq \frac{7}{2} \\ 4(7-x), \frac{7}{2} \leq x \leq 7 \end{cases}$$

$$14. \quad \varphi(x) = \begin{cases} \frac{2h}{9}x; 0 \leq x \leq \frac{9}{2}, \\ \frac{2h}{9}(9-x); \frac{9}{2} \leq x \leq 9; \end{cases} \quad \psi(x) = \begin{cases} 4x, 0 \leq x \leq \frac{9}{2} \\ 4(9-x), \frac{9}{2} \leq x \leq 9 \end{cases}$$

$$15. \quad \varphi(x) = \begin{cases} -\frac{2h}{11}x; 0 \leq x \leq \frac{11}{2}, \\ \frac{2h}{11}(x-11); \frac{11}{2} \leq x \leq 11; \end{cases} \quad \psi(x) = \begin{cases} 6x, 0 \leq x \leq \frac{11}{2} \\ 6(11-x), \frac{11}{2} \leq x \leq 11 \end{cases}$$